From John von Neumann and Oskar Morgenstern (1944) to mechanism design (2007): the never-ending and controversial story of Expected Utility Theory

Mario Fedrizzi (*)

(*) Department of Computer and Management Sciences
University of Trento, Italy

Expected Utlity Theory

- The development of Expected Utility Theory was a major intellectual achievement of the first half of 20° century
- It gave for the first time a formally axiomatized statement of what it would mean for an agent to behave in a consistent, rational matter

  (H. Simon, 1978 Nobel Prize in Econ. Sc.)

J. Von Neumann and O. Morgenstern

1944: "Theory of Games and Economic Behavior" by J. von Neumann and O. Morgenstern

"Posterity may regard this book as one of the major scientific achievements of the first half of the twentieth century. This will undoubtedly be the case if the authors have succeeded in establishing a new exact science-the science of economics." *(The Bulletin of the American Mathematical Society)*

Outline

1. Risk and uncertainty
2. The Expected Utility paradigm
3. Subjective Expected Utility
4. Prospect theory
5. SSB utility
6. Social choice
7. Qualitative decision theory

Risk and uncertainty

- The formal incorporation of risk and uncertainty into economic theory was only accomplished in 1944, when J. von Neumann and O. Morgenstern published their "Theory of Games and Economic Behavior" - although the seminal contribution of Frank P. Ramsey (1926) must be mentioned

Risk and uncertainty

- The very idea that risk and uncertainty might be relevant for economic analysis was only suggested in 1921, by Frank H. Knight in his fundamental work “Risk, Uncertainty and Profit”

- In Knight's approach, "risk" refers to situations where the decision maker can assign probabilities to the randomness which she/he is faced with

Risk and uncertainty

- On the other side, Knight's "uncertainty" refers to situations when this randomness cannot be represented using probability

- As John Maynard Keynes wrote: "By uncertain knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable"

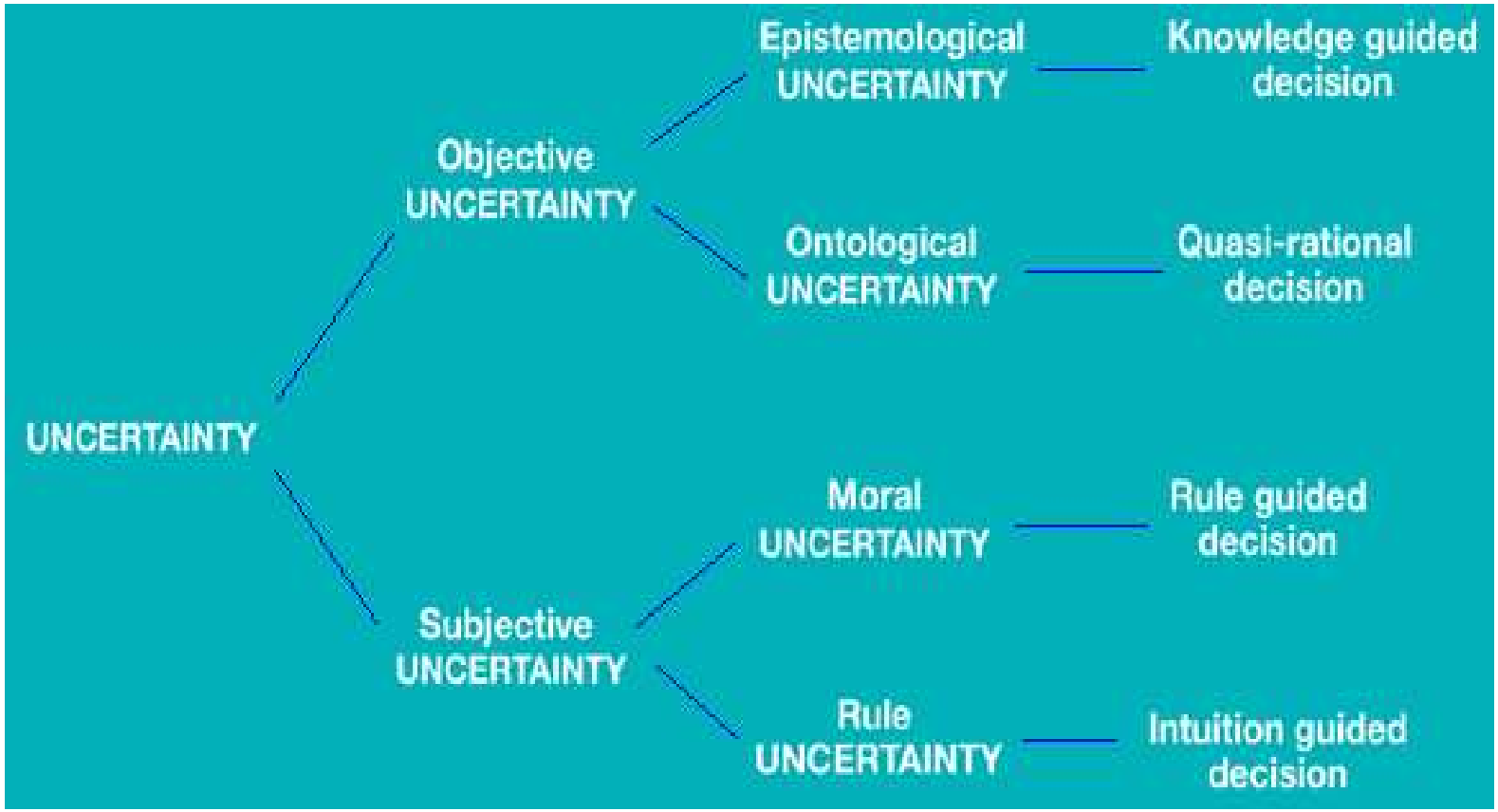
Risk and uncertainty

- The distinction introduced by Knight may be useful, in that it permits to divide theories between those which use the assignment of probabilities and those which do not make such assignments
- In this manner, the expected utility theory with objective probabilities of von Neumann and Morgenstern (1944) is one of "risk-based"

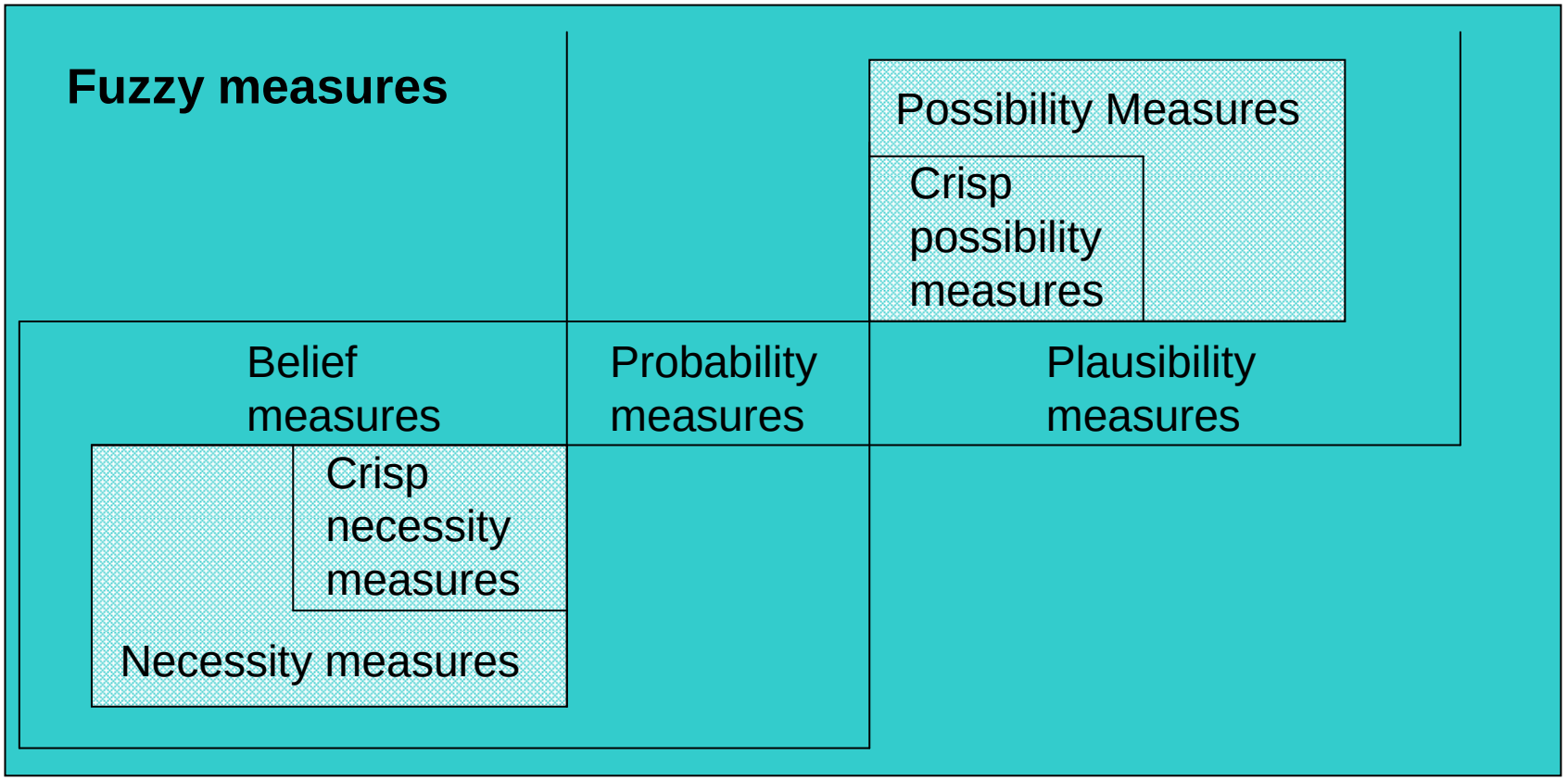
Risk and uncertainty

- The state-preference approaches of Arrow (1953) and Debreu (1959), in which there are no assignments of probabilities can be considered "uncertainty-based"
- The theory of Savage (1954), which yields expected utility with subjective probabilities, is not clearly in one camp or another

Uncertainty: an ethics-based taxonomy



Uncertainty: a measure-based picture



von Neumann-Morgenstern

The problem of rational behavior in DM

- Subject matter of economic theory is the very complicated mechanism of prices and production, and of the gaining and spending of incomes
- In the course of the development of economics it has been found that an approach to this vast problem is gained by the analysis of the behavior of the individuals which constitutes the economic community

von Neumann-Morgenstern

- One of the chief difficulties lies in properly describing the assumptions which have to be made about the motives of the individual
  - This problem has been stated traditionally by assuming that the consumer desires to obtain a maximum of utility or satisfaction
- The individual who attempts to obtain this maximum is also said to act "rationally"

Daniel Bernoulli

- "Specimen Theoriae Novae de Mensura Sortis", Commentarii Academiae Scientiarum Imperialis Petropolitanae, Tomus V, 1738

- "Exposition of a new theory on the measurement of risk", Econometrica 22, 1954

Daniel Bernoulli

- He claims that the determination of the value of an item must not be based on its price, but rather on the utility it yields
- Bernoulli states that the principle that people in risk taking situations behave such as to maximize the monetary expected value, as claimed by B. Pascal, is ill-founded

Blaise Pascal

- He laid down the principles of the theory of probability
- The theory was settled through a correspondence with Pierre de Fermat
- The correspondence arose in 1654 from a problem proposed by a gamester and related to the computation of the fair value of a two person-game

Daniel Bernoulli

St. Petersburg Paradox

- A casino allows a gambler on payment of a price P, to throw a fair coin until a head appears. If this occurs at the ith trial, the gambler wins 2^i money units
- The probability of this event is the probability of a sequence of tails on the first (i-1) trials and head on the ith trial, i.e.,

$$(1/2)^{i-1}(1/2)=2^{-i}$$

Daniel Bernoulli

- The expected winning (mathematical expectation) is infinite, therefore one should be willing to pay any sum for playing the game
- Bernoulli postulated that people would seek to maximize the expected value of the utility of the monetary value (moral expectation)

Expected Utility (EU) paradigm

- Given a convex set P of lotteries (probability distributions) defined on a set X of outcomes, each p belonging to P is viewed as a risky alternative that yields outcome x belonging to X with probability p(x) ($\Sigma p(x)=1$)
- An individual's preference relation $\geq_R$ on P is defined ($p \geq_R q$ means "p is preferred to q")
- The criterion is based on a set of statements on $\geq_R$, the so called axioms, that formalizes notions of order, independence and continuity

EU axiom 1

AXIOM OF COMPLETENESS

Given two lotteries, we assume that the order is complete (lottery stands for probability distribution)

- when p and q are lotteries, we have $p \geq_R q$ or $q \geq_R p$
- if $p \geq_R q$ and $q \geq_R p$ hold together, then we say that $p \approx q$

(≈ is the indifference relation)

EU axiom 2

AXIOM OF TRANSITIVITY:

- if $p \succeq_R q$ and $q \succeq_R r$, then $p \succeq_R r$
- We believe that choices should be transitive. This idea is part of a general argument that choice is superior when it is made as if the decision maker were bound by that choice in a broader context and across time. This argument is from Immanuel Kant (1724-1804)

EU axiom 3

AXIOM OF CONTINUITY:

if $p \succeq_R q$ and $q \succeq_R r$ then $\alpha p+(1-\alpha)q \succeq_R \beta q+(1-\beta)r$, for any α and β in (0,1)

- the rationale of this axiom is based on the observation that if α is chosen near to 1 then the difference between p and $\alpha p+(1-\alpha)q$ is negligible, therefore one can argue that $\alpha p+(1-\alpha)q$ will be preferred to q for some $0<\alpha<1$ whenever p is preferred to q

EU axiom 4

AXIOM OF INDEPENDENCE:

if $p \geq_R q$ and $0<\lambda<1$ then $\lambda p+(1- \lambda)r \geq_R \lambda q+(1- \lambda)r$ for any r in P

- probabilistically, since both distributions $\lambda p+(1- \lambda)r$ and $\lambda q+(1- \lambda)r$ yield r with probability $1-\lambda$, the preference between them depends entirely on the preference between p and q, $\lambda>0$

The fundamental theorem

- von Neumann and Morgenstern proved that the four axioms hold for $(P, \geq_R)$ if and only if (iff) a real function u on P exists such that

$$p \geq_R q \text{ iff } E[u(p)] \geq E[u(q)]$$

- They found out that u is unique up to a positive linear transformation

Theory of measurement

P. Suppes, D. Krantz, R. D. Luce and A. Tversky, *Foundations of Measurement*, Vol. I, II, III, 1971-1990, Academic Press, New York

Operational meaning: assigning numerals to more or less concrete objects according to a set of given rules

Theoretical meaning: mapping some empirical structure homomorphically into a numerical structure

Theory of measurement

<u>empirical structure</u>

$\boldsymbol{A}=\langle A;\ R_1,\ldots,\ R_n;\ f_1,\ldots,\ f_m\rangle$

<u>numerical structure</u>

$N=\langle N;\ S_1,\ldots,\ S_n;\ g_1,\ldots,\ g_m\rangle$

Theory of measurement

A mapping μ: A→N is called a representation of *A* in *N*

if it is a homomorphism, i.e.

$R_i(a_1,\ldots, a_{ki})$ iff $S_i(\mu(a_1),\ldots, \mu(a_{ki}))$

$\mu(f_j(a_1,\ldots, a_{hj}))= g_j(\mu(a_1),\ldots, \mu(a_{hj}))$

Criticisms

- According to the motto "In theory there is no difference between theory and practice, in practice there is", in late 40s and in 50s more systematic experiments were carried out, and violations of transitivity and independence were discovered

- Santa Monica Seminar (two months in the summer 1952), a group of scientists conferred on problems of decision making producing a gold mine of theoretical and empirical ideas

Criticisms

- Allais, M. (1953). Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'ecole Americaine, *Econometrica* 21, 503-546
- Ellsberg, D. (1954). Classic and current notions of measurable utility, *Economic Journal,* 64, 528-556

Criticisms

- Edwards, W. (1954). The theory of decision making, *Psychological Bulletin*, 51, 380-417
- Edwards, W. (1961). Behavioral decision theory, *Annual Review of Psychology*, 12, 473-498
- These papers cover, through April 1960, the psychological and economic theories of riskless and risky decision making, and the experiments relating to these theories

Subjective probability

The subjective theory of probability was developed by de Finetti and Ramsey in the late 1920s and 1930s

They formalized the concept of choice-based subjective probability assuming that individuals seek to maximize expected utility when betting on the truth of propositions

Subjective probability

In de Finetti's approach, the probability of an event is the price at which an individual is willing to pay for a lottery ticket that yields 1 unit of money if the event occurs and nothing otherwise

de Finetti introduced the notation "Pr" to refer indifferently to Probability, Price, and Prevision, treating them as labels for a unique concept

Sujective Expected Utility

- While synthesizing the ideas of de Finetti and von Neumann and Morgenstern, Savage introduced conditions that are necessary and sufficient for the existence and uniqueness of utility and probability, and the characterization of individual choice as expected utility maximizing behavior

Savage, L. J. (1954). *The Foundations of Statistics*. J. Wiley and Sons, New York

Subjective Expected Utility

Components of his model:

i) a subjective probability measure *p* defined on the subsets (events) of a set *S* of states of he world

ii) a utility function *u* defined on a set *X* of decision outcomes

iii) seven postulates on the preference structure

Prospect theory

- In response to the empirical violations of the Expected Utility paradigm, a number of alternative models of risk preferences and beliefs have been developed, most of which replaced the expected utility formula with alternative formulas that individuals are assumed to maximize
- The earliest of these models, proposed by Ward Edwards, were adopted by Daniel Kahneman and Amos Tversky in their "prospect theory"

Prospect theory

- Kahneman, D., Tversky, A. (1979). Prospect theory of decisions under risk. *Econometrica*, 47, 263-291
- Kahneman and Tversky have shown that weakening the independence axiom and retaining the assumptions of order and continuity, the expected utility criterion possesses a better explanatory power for economic phenomena that improves the one of the traditional approach

Prospect theory

- Certainty effect:
  - people underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty. This behavior generates risk aversion in choices involving sure gains and risk seeking in choices involving sure losses

Prospect theory

- Isolation effect:
  - people usually discard components that are shared by all prospects under consideration. This tendency leads to inconsistent preferences when the same choice is presented in different forms

Prospect theory

- An essential feature of the prospect theory is that the carriers of value are changes in welfare, rather than final states. This assumption is compatible with the principle that our perceptual apparatus is attuned to the evaluation of changes rather than to the evaluation of absolute magnitudes

Prospect theory

- Value should be treated as a function in two arguments: the asset position that serves as reference point, and the magnitude of the change from that point
- The value function is usually concave for gains and convex for losses

Prospect theory

- Prospect theory was one of the first models for decision under risk that permitted descriptive deviations from rationality and achieved theoretical tractability at the same time
- However, violations of stochastic dominance are implied
- The problem has been solved by cumulative prospect theory

Prospect theory

- Kahneman, D., Tversky, A. (1992). Cumulative prospect theory
- They invoked the Quiggin's idea of rank-dependent utility (1981) combining the descriptive advantage of original prospect theory with the theoretical advantages of rank-dependent utility
- Cumulative prospect theory provides one of the most promising non-expected utility models available

Prospect theory

- Cumulative prospect theory not only satisfies stochastic dominance, but it also gives a better account for a number of empirical findings

- Fennema, H. and Wakker, P. (1997). Original and cumulative prospect theory: a discussion of empirical differences. *Journal of Behavioral Decision Making,* 10, 53-64.

Prospect theory (?)

- Our experimental evidence indicates that the best characterization of subjects' behavior is given by the class of reverse S-shaped functions, as suggested by Markowitz, which is just the opposite of the S-shaped function advocated by prospect theory (M. Levy, H. Levy (2002). Prospect theory: much ado about nothing? *Management Science*, 48, 1334-1349)

Bounded rationality

Simon, H. A. (1957). Models of Man. John Wiley and Sons, New York

March, J. C., H. A. Simon (1958). Organizations. John Wiley and Sons, New York

Simon, H. A. (1979). Rational decision making in business organizations. *The American Economic Review,* 69, 493-513

SSB utility theory

- Fishburn, P. C. (1982). Nontransitive measurable utility. *Journal of Mathematical Psychology*, 26, 31-67
- Fishburn, P. C. (1984). SSB utility theory: an economic perspective. *Mathematical Social Sciences*, 8, 63-94
- Transitivity and independence are dropped

SSB utility theory

- A rational economic person compares two outcomes x and y from X, assuming that she/he prefers one to the other or is indifferent between them
- A cardinal magnitude φ(x,y) associates to each ordered pair (x,y) in XxX the intensity of her/his preference for x over y

Social choice theory

Two distinct sources:

I) Normative analysis of personal welfare (Bentham, 1789)

II) Mathematical theory of elections and committee (Borda, 1781; Condorcet, 1785)

- The union of the two sources took the form of the "Impossibility theorem" (Arrow, 1951)

Social choice theory

- Arrow (1951) introduced the so-called Social Welfare Function (SWF) as a functional relation defining a social preference relation R starting from the individual preferences estimated by the members of a group.

$R_1,\ldots, R_n$ individual preference relations

X set of alternatives (social states)

$R=F(\{R_k\}$ SWF

Social choice theory

Axiom 1 (Unrestricted domain)

- The domain of F contains all possible n-tuples of individual preferences

Axiom 2 (Weak Pareto principle)

- If xP_ky (k=1,…, n), then xPy for any x, y in X

Social choice theory

Axiom 3 (Non-dictatorship)

- There is no k such that for all n-tuples in the domain of F and for all pairs (x, y) in X if xP_ky, then xPy

Axiom 4 (Independence of irrelevant alternatives)

- If for any subset S of X, all the individual preferences remain the same for every pair of alternatives from S, then the choice set of S should remain the same too

Social choice theory

- Arrow's Impossibility Theorem: there exists no SWF satisfying axioms 1, 2, 3,

- Arrow, K. J. (1963). Social Choice and Individual Values. J. Wiley and Sons, New York (originally published in 1951)

Social choice theory

Fishburn, P. C. (1973). *The Theory of Social Choice.* Princeton University Press, Princeton

Harsanyi, J. (1977). *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*. Cambridge University Press, Cambridge

Sen, A. K. (1977). Social choice theory: a e-examination. *Econometrica*, 45, 53-89

Kelly, J. S. (1978). *Arrow Impossibility Theorems*. Academic Press, New York

WGDSS and Arrow's theorem

Now, internet and web technologies allow the development of distributed DSS to provide support to groups who may seldom or never meet and take their decisions implementing processes mediated through time and space by the technology

How does the Arrow's theorem influence the design of WGDSS?

Simon French (2004) "Web-enabled strategic GDSS, e-democracy and Arrow's theorem: a bayesian perspective"

Game theory

- A player is assumed to have preferences over lotteries that are generated by her/his mixed strategies and by mixed strategies of the opponents
- As long as the player do not violate the axioms of von Neumann and Morgenstern, she/he can be described as if she/he is maximizing the expected value of an appropriate chosen utility function

Portfolio selection

- After the axiomatization of expected EU paradigm, economists began immediately seeing the potential applications of EU to economic issues like portfolio choice, insurance, etc. (H. Markowitz)
- Certainty-equivalent, risk-premium, risk-aversion measures were introduced by J. Pratt and K. Arrow

Fuzzy (possibilistic) decision theory

Zadeh, L. H.(1973), "Outline of a new approach to the analysis of complex systems and decision processes", *IEEE Transactions on Systems, Man and Cybernetics*, *SMC-3, 1*, 28-45

Billot, A. (1995), "Fuzzy decision theory", in Decision under Uncertainty, Int. School of Econ. Res., Certosa di Pontignano, Siena, Italy, 39-75

Qualitative decision theory

- A counterpart to von Neumann and Morgenstern EUT was proposed in the framework of possibility theory by D. Dubois and H. Prade in

  Possibility theory as a basis for qualitative decision theory (1995). *Proc. of the 14° Int. Joint Conf.on Artificial Intelligence (IJCAI), Montreal*

Qualitative decision theory

- A utility function, representing a preference ordering among possibility distributions, is established
- Possibilistic utility obeys a set of axioms pertaining to decision-maker's behavior, and is tailored to reasoning under incomplete states of knowledge
- Only max, min and order-reversing operations are used on the scale

Mechanism design theory

- Engineering side of economic theory
- We begin by identifying our desired outcome or social goal
- We then ask whether or not an appropriate institution (mechanism) could be designed to attain that goal
- A mechanism is an institution, procedure, or game for determining outcomes

Mechanism design theory

- Maskin, E. and T. Sjöström (2002). Implementation theory, in K. Arrow, A. Sen, and K. Suzumura, (eds.), *Handbook of Social Choice and Welfare*, Vol. 1, Elsevier, 237-288
- 2007: Nobel Prize awarded jointly to L. Hurwicz, E. Maskin, and R. Myerson "for having laid the foundations of mechanism design theory"

Nobel prizes

Kenneth Arrow (1972), *General economic equilibrium: purpose, analytic techniques, collective choice*

Herbert Simon (1978), *Rational decision making in business organizations*

Maurice Allais (1988), *An outline of my contributions to economic science*

Nobel prizes

Harry Markowitz (with M. Miller and W. Sharpe, 1990), *Foundations of portfolio theory*

John Harsanyi (with J. Nash and R. Selten, 1994), *Games with incomplete information*

Amartya Sen (1998), *The possibility of social choice*

Nobel prizes

Daniel Kahneman (2002), *Maps of bounded rationality: a perspective on intuitive judgment and choice*

Leonid Hurwicz (with E. Maskin and R. Myerson, 2007), *Mechanism design: how to implement social goals*