Some Experimental Results for a Non-Equilibrium Bargaining Model of War

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August 8, 2008

Abstract

We provide the first experimental results on a two-stage model of militarized conflict based on the ubiquitous ultimatum bargaining game that dominates much of the theoretical analysis of international relations. We illustrate how data from experimental trials deviate from the equilibrium predictions from both a complete information version of the model and the quantal response version of the model. We discuss how the results from the experiments can inform future theoretical, empirical, and experimental work on bargaining theories of war.

Bargaining models have become commonplace in the international conflict literature in the past two decades, and have provided a number of significant insights regarding when and why states resort to arms. The bargaining model has been used to explain how wars begin, how wars end, and how long we can expect wars to last. In addition the model has been adapted to explain the durability of peace agreements, extended deterrence, territorial conflict, trade and conflict, and the democratic peace. The theoretical transport of this simple model is quite impressive.

Like most formal models in political science, the bargaining model of war is an equilibrium model. That is, scholars use the bargaining framework to derive equilibrium solutions in order to make statements about how players make demands, and either avoid or enter conflict. Game theoretic models in general produce these sharp predictions but have little to say about when or why players might deviate from equilibrium strategies. This is true of the bargaining model of war as well, and has attracted a significant amount of scholarly attention, mainly aimed at refining rationalists theories of war so they can explain why states cannot find negotiated solutions and instead end in conflict, which is taken in most models to be ex post inefficient. The puzzle of non-equilibrium behavior is portrayed acutely in laboratory experiments in economics which widely demonstrate that players often diverge from equilibrium strategies; these sorts of findings helped inspire equilibrium concepts that admit errors and even learning processes in decision making. Among these are McKelvey & Palfrey's (1995) influential quantal response equilibrium, and evolutionary game theory.

Resutls from our own laboratory experiments echo these concerns. We designed and ran experiments based explicitly on the two-stage bargaining model of war. A key aspect to the experiment was that information regarding allocations, costs, demands, and probability of success in the event of a conflict was perfect information for all participants. Despite this information, conflict emerged in a significant percentage of the cases. This feature indicates an important aspect to the bargaining model. If conflict can occur in a perfect information setting, then efforts to explain the occurrence of conflict observed empirically by appealing to extensions such as assymetric information are overlooking problems in the bargaining model itself. If subjects engage in conflict in a perfect information setting, then the bargaining model, as specified, must miss some aspect of their decision-making process.

What we propose in this paper is a modeling strategy that generalizes the bargaining model in an attempt to better capture decion-making behavior. In particular, we use McKelvey and Palfrey's (1995) quantal response equilibrium concept, which allows us to introduce noise into the model. Specifically, the standard bargaining model assumes that players are infinitely 'rational' - where 'rational' is defined as infinitely responsive to expected payoffs. The quantal response extension of the model relaxes the assumption of perfectly responsive players, allowing noise into the decision-making process. This noise is represented by a parameter, λ, which can range from zero - random behavior - to infinitely large values - perfectly responsive. This extension is useful in that it allows us to explain the occurrence of conflict in a perfect-information setting. Furthermore, by allowing λ to vary, we can compare QRE results for different levels of λ to our experimental results to identify which values yield a good fit for our data. With regard to particular constraints, in the international context, on the responsiveness of decision makers to expected payoffs, we argue that our model more broadly characterizes the imperfections in strategic decision making, and permits more refined predictions about behavior than most standard bargaining models do.

To accomplish this task, we begin with a simple bargaining model where conflict is the outside option, and derive the standard Nash equilibrium solution. Drawing on insights from experimental economics specifically on "noisy introspection," we characterize that equilibrium as a special case arising when each side believes the other to be perfectly rational. Again, by "perfectly rational," we specifically mean that one side believes the other is completely capable of maximizing, given the conditions of the game (e.g. information asymmetries, etc.); in other words, there is no noise or uncertainty surrounding a player's beliefs about her opponent's maximizing ability. In the context of a quantal response model, we then show that the Nash equilibrium is the special case where both sides share this belief, and that expected behavior diverges from that equilibrium when players have different beliefs about their opponents' abilities to maximize.

A brief and simple example will set the stage for our discussion of the technical aspects of the model, of how players develop beliefs about their opponents' maximizing skills, and of how we attach this to world politics and problem of war. Suppose two poker tournaments, the first strictly among championship players, the second, a sort of pro-amateur tournament where skilled players play against novice players. In the first, two championship poker players face each other in a poker tournament. Each knows the other to be an outstanding, experienced, and skilled player, and therefore, each player has a significant set of expectations about what responses to expect. In this case, the outcome of each hand will be influenced by the chances given by the random draw of cards, by each player's beliefs about his opponent's cards, and by the stakes or costs of each hand. Neither player has any reason to question whether his opponent will play a "best response" strategy, nor whether his opponent knows which strategy is a "best response."

Now, for contrast, imagine the pro-am tournament, and suppose one of these championship poker players enters a game against a novice, who knows the rules of poker and has played, but has played very little. The outcome of the game will be influenced by the same set of factors as in the first tournament, but will also be shaped by the champion's belief that his opponent will not always know a "best response" when he sees it, nor that he will play that "best response" strategy consistently. The champion will develop an estimate, but one surrounded by noise and uncertainty, of the chances his novice opponent will play maximizing strategies, and he will shape his own strategies based on that belief. For instance, whereas the champion will not overbid in his game against the other champion player, knowing that opponent would recognize his strategy and counter it effectively, he might have incentive and opportunity to overbid against the novice based on his belief the novice is not a perfect maximizer and may not recognize the strategy, and may not adopt a "best response." In this case, the outcome of the game is influenced by a set of beliefs developed through noisy introspection, learning, but in a noisy environment. Key to this example is that the standard set of factors deemed influential in poker are constant across these two different tournaments. The only shift is what a player believes about his opponent's maximizing ability or rationality. Importantly, the result is a change in strategy away from the equilibrium one might derive in the championship tournament. Deviations from equilibrium are not strictly because one player is not skilled; they arise because the skilled player believes his opponent is not skilled and adapts his strategies appropriately to exploit that very fact.

Let us now connect this example to the formal aspects of the model and to the standard bargaining model of war. Game theoretic models, even those admitting imperfect decision making and errors, would arrive at an equilibrium prediction of the strategies the two sides choose and would not be able to predict deviations from equilibrium. In the professional tournament example, the model would probably predict relatively well, deviations from equilibrium principally arising from induced uncertainty (bluffing), or more generally from information asymmetries in the game itself. In the pro-am example, standard bargaining models would predict less well because there would likely be more deviations from the equilibrium and many of those would be due to beliefs about players' maximizing abilities derived from noisy introspection. The quantal response approach we propose, on the other hand, would describe the probability distribution surrounding equilibrium where deviations are explicitly a function of the beliefs each side develops about the other's maximizing abilities. This discussion points to our first challenge. The model we derive must capture the Nash equilibrium, must characterize the probabilities of strategies around the equilibrium, and must attach those probabilities to beliefs distinct from those in the bargaining model itself (i.e. information asymmetries about costs, chances of winning, etc.).

The bargaining model of war elaborates a host of mechanisms by which beliefs influence strategic choices, but always having to do with beliefs about factors inside the bargaining structure, like relative power, costs for war, and the like. The bargaining model of war does not admit beliefs about other states' skills in bargaining or maximizing, nor does it explain what appear to be non-equilbrium strategies except those resulting from incorrect beliefs inside the bargaining structure. Our model will account for these things, but presses on us a second challenge. We must explain how and why it is reasonable in the story of interstate interactions to believe that states are not perfect maximizers, and what observable variables in world politics are likely to shape what one state believes about another state's ability to maximize. In large part, this discussion is reserved for another paper, but some of the answers lie in interstate experience and in the infrequency with which states bargain in crises. That inexperience leaves states to develop expectations about whether their opponents are good or bad at maximizing based on what they know or believe about like-states, for example, other authoritarian states, other states with similar cultural or religious characteristics, etc.

In the section that follows, we discuss the contributions derived from the standard bargaining model of war and elaborate the rational for a quantal response approach founded in incorporating states' beliefs about each others abilities to maximize in bargaining. After that, we produce a simple, complete information bargaining model solved by backwards induction, and then extend that model in a quantal response context to describe the probabilities associated with demands of different magnitudes, and the probability of conflict given those demands. In doing this, we account for variability in states beliefs about how well their opponents maximize. Finally, we design a laboratory experiment directly from the bargaining model and evaluate the experimental results relative to Nash equilibrium expectations and quantal response expected probabilities.

Bargaining and War

A large part of the world politics literature addresses the problem of war. Formal models of militarized conflict are often rooted in the idea that states are, in fact, bargaining over the division of some good or goods, and that the economic bargaining model can be usefully applied to the problem of war. Schelling (1960) is usually credited with originating the notion that states bargain in this fashion. Until the late 1980s, most formal work in international relations did not examine sequential bargaining under incomplete information. The literature almost exclusively favored simultaneous move games (e.g. Snyder & Diesing 1977), games under complete information (e.g. Brams & Kilgour 1988), and expected utility (non-interactive) models (e.g. Bueno de Mesquita 1981). Morrow (1989) presents one of the first incomplete information bargaining models of (non-nuclear) conflict with sequential moves in the international relations literature. Since Morrow's paper, bargaining models have become ubiquitous in the conflict literature, and have produced a large number of important insights regarding bargaining and war. Those insights address the outbreak of war (e.g. Morrow 1989, Fearon 1995, Gartzke 1999), mechanisms for signaling (e.g. Fearon 1994, Schultz 1998, Fearon 1997, Slantchev 2005), alliance behavior (e.g. Morrow 1991, 1994; Smith 1995, 1996, 1998), war termination and continuation (e.g. Filson and Werner 2002, 2004; Wagner 2000), domestic politics and war (e.g. Bueno de Mesquita & Lalman 1992, Bueno de Mesquita, Morrow, Siverson & Smith 1999, Powell 1999), and implications regarding the distribution of power, conflict and endogeneity of demands (e.g. Powell 1999).

Morrow (1989) and Fearon (1995) produced what are among the most influential papers on bargaining and war, perhaps because both point to crucially important elements of the bargaining model that imply challenges for the scholarly community. Morrow argues that bargaining models imply strategic selection and the importance of beliefs. These collectively produce an empirical challenge to model strategic, non-random selection and to measure elements of beliefs; both of these pose significant empirical difficulties. Fearon argues that rationalist models inadequately deal with the problem that both sides in bargaining have incentives to misrepresent themselves (principally their capabilities), thus increasing the chances states disagree over the distribution of power and thus, the chances of war. The challenge to scholars of conflict is to devise theoretical mechanisms by which states can solve this problem and arrive at negotiated solutions, avoiding the outside option of war.

Jointly, these challenges have shaped much of the literature on bargaining and war. It is still the case, however, that empirical evaluations of the bargaining model are relatively infrequent due mainly to the observability problem. For instance, if a challenger's beliefs about a defender's costs for war are influential in determining whether the challenger makes a demand and the size of the demand, empirical models would require measures of those beliefs, and of the nature of demands. Further, despite significant development of signaling arguments that suggest answers to Fearon's challenge, the literature is troubled by the outbreak of war and the inability of states to find mutually preferred negotiated outcomes. So even when states can signal by tying hands or sinking costs, war still occurs.

We propose another pathway by which conflict occurs, specifically that states' leaders are uncertain about each others' abilities to maximize, or more simply, about their opponents' levels of rationality. The standard economic bargaining model and the same model with respect to war assume actors are completely rational, and that they believe each other to be completely rational. As such, the standard bargaining model is an extreme case of the quantal response model in which λ is infinitely large. We relax this assumption. Doing so allows us to account for the prospect that in international negotiations, as compared to in economic transactions in the market, there is a difference in the frequency and history of interactions. Economic markets are typically characterized by large numbers of interactions between buyers and sellers who negotiate exchanges repeatedly. Even if a consumer buys some particular good infrequently, she is still informed by the large body of common knowledge regarding the transaction, because such transactions are so commonplace. The international system, on the other hand, is characterized by episodic interactions varying in intensity, and cumulating in the infrequent necessity of crisis bargaining. When buyers and sellers agree on a price, they do so in the shadow of long experience. When states bargain in crises, they do so in the shadow of their own characteristics and the bargaining structure itself, but with little in the way of experiential guidance regarding the transaction of war. This is necessarily so given the costs of war. This difference between bargaining in the market and bargaining in crises points to the potential inadequacy of standard bargaining models in describing interstate behavior, because inexperience in crisis bargaining implies states are likely to be less than perfect maximizers. Standard models therefore require that states and their leaders are skilled maximizers with complete understanding of bargaining. Moreover, these models require that each side believes the other is completely rational in this sense, and makes strategic choices founded in that belief. Metaphorically, the assumption is that states are professional poker players; each knows the other to be strategically skilled, so states should choose strategies as if they are bargaining with fully maximizing opponents. Because of this restriction of the models, equilibrium decisions are deterministic and are perfectly anticipated by other players, and hence, deviations from the equilibrium path are not accounted for.

More plausibly, leaders are hobbled in their capacities to behave maximally by their own inexperience (after all, leadership tenure is relatively short regardless of regime, by agency loss due to their advisory structures (Kher N.d.), or by the relative rarity of interstate crises from which leaders have the chance to learn. Since 1918, the average leader around the globe has survived approximately just under five years in office; the median survival is four years. The most authoritarian leaders in that period enjoyed about 11 years in power, while the most democratic survived just under four years in office on average. The average leader did not take part in an interstate military crisis (the median number of crises per leader is zero, the mean is .5).[1] At a minimum, the prima facie evidence indicates little reason to believe states' leaders have extensive experience guiding the ship of state or that they have frequent experience bargaining with other states where war is the outside option.

Even institutional memory is hobbled by the rarity of bargaining in the shadow of war. As a result, factors outside the bargaining framework influence the decision making capacities of leaders. As poker players develop opinions about their opponents' abilities to

[1] We use the Archigos data on leaders, Polity IV data on regime characteristics, and Interstate Crisis Behavior (ICB) data to construct these characterizations of states' leaders.

maximize, so will leaders develop views regarding the maximizing capacities of their bargaining opponents. Notice that these views about their opponents are likely to shape their own strategies, and arise separately from factors normally asserted to shape bargaining like information asymmetries. Absent specific and repeated experience with war, it is difficult to imagine how leaders can adequately or precisely estimate or comprehend the costs of war. For themselves, leaders will often suffer some form of organizational instinct to emphasize optimistic assessments and to deemphasize negative reports (Levy 1986). Authoritarian leaders might suffer the so-called "autocrat's dilemma," unable to assess their own chances in war or costs for war due to their subordinates fears of retribution for reporting bad news, or desire to please the leader (Wintrobe 2000). The same sort of dynamic is widely reported in historical accounts of US decision making in Vietnam. So absent experience in leading the state into war, leaders are not likely able to maximize very effectively, both due to their own inexperience evaluating the problem of war, and because of the information-distorting environment around them. As a stylized illustration, consider the case of the French General Staff's "Plan 17" at the outset of the First World War. Via observation of German preparations and accounts by French intelligence, the Deuxieme Bureau, from spies behind German lines, the French held solid intelligence that the Germans would employ enormous numbers of reserve troops in fighting on the western front. This information was significant because the French were debating between defensive and offensive strategies. If the Germans were to use their reserves, no offensive strategy could possibly succeed, and would likely divide French forces disastrously. The French General Staff discounted all the evidence and selected an offensive strategy which the Germans easily defeated because of the superior numbers of troops they fielded using their reserves. In contrast to the French military's disregard of intelligence, the German military acted on its intelligence facilitating its route of the French (Tuchman 1962). Both sides in this case had ample access to information regarding p and c but used that information to different degrees of effectiveness.

The standard sequential bargaining model provides a sharp prediction of a demand that is accepted immediately, with no conflict. The stark predictions of this model can be enriched by injecting noise in decision making, while maintaining the equilibrium requirement that distributions representing beliefs match the probability distributions of decisions (a quantal response equilibrium). This equilibrium approach has been used with successs in explaining behavior in long series of interactions in political participation games (Goeree & Holt 2005). The equilibrium approach, however, may not be appropriate in games played infrequently or only once, i.e. belief distributions may be too dispersed in the sense that the noise parameter in beliefs is different from the noise in actual decisions. Even worse, there may be systematic biases caused by overconfidence or selective reliance on information that is too favorable (e.g. Camerer 2002). This dynamic seems present in the French case described above.

It is, of course, possible to generalize the notion of equilibrium by introducing "noise" via probabilistic choice models, and our work is influenced by this approach.[2] The self-fulfilling nature of beliefs in equilibrium models (with or without noise) can be defended in situations where players have had a chance to learn from extensive experience, which is often the case in repeated market interactions. Although there may be multiple reasons for non-equilibrium behavior either in the political or economic interactions, or in laboratory experiments, we focus modeling beliefs in the absence of extensive experience or learning opportunities. In such cases, learning may arise from introspection about others' behavior in the context of the payoff incentives of the game. Moreover, learning may be systematically biased by factors like overconfidence, anchoring and adjusting from the status quo, selective confirmation of prior beliefs, and misperceptions of others' levels of rationality.

Drawing on insights from behavioral game theory, we posit that interactions between states are probably not best characterized by strict Nash equilibria, insofar as such outcomes rely on decision makers having significant experiences with the outcomes of interest, a requirement that does not transfer particularly well to modeling the problem of war. As Schelling (1960, 117) points out, "there are undoubtably special cases in which one can sup-

[2]Also see Morton (1999) for a good discussion of deterministic equilibrium models, the zero likelihood problem and the quantal response solution due to McKelvey and Palfrey (1995).

pose that the other player is like one's self in basic values and can consequently estimate the other's values by the simple application of symmetry. But in too many exciting cases, one plays an opponent who is a wholly different kind of person." Leaders and states vary widely and are often little-known to their bargaining opponents in the particular context of the payoff incentives related to crisis bargaining. Unlike the market context where interactions are predictable by their repetition such that the variance in bargaining behavior is limited and informed by experience, the problem of bargaining where war is the outside option is sufficiently rare that one does not know whether one's opponent is wholly different or not. Put another way, rationality - or responsiveness to expected payoffs - of one's opponent is probably in doubt, is certainly unknown, and is possibly assessed introspectively. Further, we propose that standard models in general, by their strong assumption regarding the 'rationality' of all players, are broadly inadequate for describing interstate crises and war. The model we propose is characterized by probabilistic "quantal" responses, which admit distributions of outcomes (e.g. McKelvey & Palfrey 1995, McKelvey & Palfrey 1998, Signorino 1999). Moreover, the unique, non-repeated nature of most conflicts between countries may make it undesirable to make such strong assumptions regarding the absence of noise. The very process of belief formation is likely to be noisy, and subject to bias.

Our starting point, the quantal response equilibrium (QRE), follows the Goeree and Holt (2000) analysis of multi-stage bargaining in economics and Signorino's (1999) application to the bargaining model of war. These applications use the QRE concept as a mechanism for producing a statistical model that is a generalization of the formal bargaining model.[3] The QRE forms the link between formal and statistical models by incorporating uncertainty via

[3]There have been several applications of the QRE model specification to the quantitative study of crisis bargaining (Signorino 1999, Signorino and Tarar 2006). The efforts focus on testing the complete game theoretic model rather than a subset of comparative statics (Signorino 2007, Carrubba, Yuen, and Zorn 2007). These applications fix the noise or rationality parameter λ and attempt to obtain an estimate of the effect of the variables that make up the actors' utility functions. Although these applications have proven fruitful, there is a lively debate about how to specify the statistical analog of the QRE. Addressing uncertainty and updating in the theoretical models and the related implications for statistical analysis are an important part of this debate (Signorino 2003, Lewis and Schultz 2003, Schultz and Lewis 2006, Wand 2006).

an error structure into the game. Goeree and Holt and others have found experimental evidence of "inequity aversion" to unequal earnings, which might be modified in the political context to include a type of loss aversion to deviations from the *status quo*.

The argument we put forward represents a significant departure from the Goeree and Holt (2000) and Signorino (1999) QRE models by shifting focus away from the equilibrium requirement that beliefs match distributions of actions. In the context of the quantal response structure, our interest is on the ancillary parameter denoted λ that ranges from 0 (full noise or zero rationality) to infinity (perfect rationality). In a quantal response equilibrium model, the same λ parameter is used to determine the noise in a player's own decisions and the noise that a player believes is characteristic of another player's decisions. As noted above, equilibrium assumptions may be less relevant in one-shot strategic interactions, and in these cases belief distributions may be more diffuse than choice distributions. As a result, we do not fix the noise parameter, but rather, we treat it as a factor that may differ with experience across levels of thinking about what others will do or what they think as explained below.

In the following pages, we present a simple bargaining model of conflict and derive quantal response results for the model. The formal model informs our experimental design, and we relate our experimental results to Nash predictions and to the QR predictions.

A Bargaining Model of Conflict

In our model, two leaders negotiate how to split a parcel of territory, which is normalized to be of size 1. The status quo division is $(q, 1-q)$, where q is the challenger's initial allocation of the territory. In the experiment, the challenger was termed the "proposer" and the allocations were measured in dollar amounts. The proposer begins by demanding x and offering $1-x$ to the "responder." The responder can either accept the offer, in which case it is implemented, or make a counteroffer to divide the discounted value of the second-stage "pie," δ, where $0 < \delta < 1$ is a discount parameter that penalizes delayed settlements. Thus the counteroffer is a split of $\delta - x_2$ for the original proposer and x_2 for the responder. If the responder makes such a counteroffer, then the proposer must decide whether to accept the counteroffer or to engage in conflict to gain control of all of the territory. As before, the proposer will win this conflict with probability p, which was explained to subjects in the experiment in terms of throws of a ten-sided die. The costs of conflict are c_P for the proposer and c_R for the responder. The expected payoffs with conflict are: $\delta p - c_P$ for the proposer and $\delta(1 - p) - c_R$ for the responder.

Assuming that both the proposer and the responder are fully informed, rational, prospective decision makers, we can solve this game through backward induction. The resulting subgame perfect Nash equilibrium involves an initial demand of $x = 1 - \delta + \delta p - c_P$ by the proposer that is immediately accepted. The prediction is that there will be no conflict, but behaviorally, we know conflict does occur. Moreover, our experimental results show that even in this simple setting of complete information, with none of the auxiliary assumptions described above (incomplete information, reputation building) we see out-of-equilibrium conflict. If this is the case, we need a closer inspection of the simple bargaining model.

The problem in crisis bargaining with any equilibrium concept is that actors cannot be fully rational in the sense of completely maximizing, because of their inexperience with bargaining where war is the outside option. Consider this on two dimensions. Even the most belligerent states in war-prone regions rarely fight wars, and also rarely bargain where war is the outside option. Further, belligerent states are often piloted by leaders who have relatively limited experience in conducting international affairs. The result is that decision makers are uncertain about the rationality of their opponents. Imagine playing a bargaining game, for instance, against an opponent who is believed to make choices by tossing a coin. The strategy one chooses against such an opponent is certainly different from the strategy one chooses against an opponent believed to be fully rational. That “noise” produces errors in bargaining which result in experimental data in deviations from Nash.

Here, we offer a model that explicitly addresses players’ perceptions of the levels of rationality of their opponents, and incorporates it in understanding how this perception affects demands and responses. We adopt an approach where we calculate the quantal response equilibrium for a common λ (expected rationality of one's opponent) and look at the predicted behavior for large values of λ, i.e. as behavior becomes fully rational in the limit. This approach also permits us to derive numerical predictions about how the probability of conflict varies with the balance of power, as measured by the proposer win probability, p. Then we will discuss how to generalize this model to allow different levels of rationality, i.e. to allow players to attribute lower levels of rationality to the other player. This approach is consistent with arguments on noisy introspection that incorporate players' beliefs about their opponents' levels of rationality (e.g. Goeree & Holt 2000, Goeree & Holt 2004)

To incorporate noise into the model and produce predictions the deviate from the no-conflict equilibrium, we present a QRE version of the model. In the second stage, the responder must evaluate expected payoffs from demanding x_2. This expected payoff is the demand x_2 times the probability that it is accepted (first term in square brackets below) plus the responder's expected payoff from conflict, $\delta(1-p)-c_R$, times the probability of rejection (second term in square brackets below):

$$\pi_R(x_2) = x_2\left[\frac{e^{(\delta-x_2)\lambda_P}}{e^{(\delta-x_2)\lambda_P}+e^{(\delta p-c_P)\lambda_P)}}\right]+(\delta(1-p)-c_R)\left[\frac{e^{(\delta p-c_P)\lambda_P}}{e^{(\delta-x_2)\lambda_P}+e^{(\delta p-c_P)\lambda_P)}}\right] \tag{1}$$

Notice that the probabilities of acceptance and rejection are ratios of exponentials of the proposer's payoffs for acceptance and conflict, and that these probabilities depend on λ_P, the proposer's rationality parameter. Next, the probability that the responder will select any counter-demand x_2 is a ratio of exponentials of the expected payoffs for various counter-demands, each multiplied by the responder's rationality parameter:

$$Pr(x_i) = \frac{e^{\pi_R(x_2)\lambda_R}}{\sum_j e^{\pi_R(x_{2j})\lambda_R}} \tag{2}$$

where the sum in the denominator is taken over the various possible counter-demands that the responder may consider. The responder's expected payoff (REP_2) for the second stage, then is the summation of the expected payoffs for each x_2, multiplied by the probability that the responder demands that level of x_2: $REP_2 = \sum \pi_R(x_2)Pr(x_2)$.

Similarly, the proposer's expected payoff for a given counter-demand x_2 received is the proposer's share, $\delta - x_2$, times the proposer's own acceptance probability, plus the proposer's conflict payoff, $\delta p - c_P$, times the conflict probability:

$$\pi_P(x_2) = (\delta - x_2)\left[\frac{e^{(\delta - x_2)\lambda_P}}{e^{(\delta - x_2)\lambda_P} + e^{(\delta p - c_P)\lambda_P}}\right] + (\delta p - c_P)\left[\frac{e^{(\delta p - c_P)\lambda_P}}{e^{(\delta - x_2)\lambda_P} + e^{(\delta p - c_P)\lambda_P}}\right] \tag{3}$$

The proposer's expected payoff (PEP_2) for the second stage is the sum of these expected payoffs for each x_2, multiplied by the probability that the responder demands that level of x_2: $PEP_2 = \sum \pi_P(x_2)Pr(x_2)$. The second stage feeds the analysis for the first stage, where the proposer's expected payoff for an initial demand x is the product of x and the probability that it is accepted, plus second-stage payoff PEP_2 times the probability that the initial demand is rejected:

$$\pi_P(x) = x\left[\frac{e^{(1-x)\lambda_R}}{e^{(1-x)\lambda_R} + e^{(REP_2)\lambda_R}}\right] + (PEP_2)\left[\frac{e^{(REP_2)\lambda_R}}{e^{(1-x)\lambda_R} + e^{(REP_2)\lambda_R}}\right] \tag{4}$$

Notice that the probabilities of acceptance and rejection are ratios of exponentials of the responder's payoffs for acceptance $(1 - x)$ and rejection, which triggers the second stage and a payoff of REP_2. These probabilities depend on λ_R the responder rationality parameter.

Finally the probability that the proposer chooses any given initial demand x is a ratio of exponentials of the associated expected proposer first-stage payoffs:

$$Pr(x) = \frac{e^{\pi_P(x)\lambda_P}}{\sum_j e^{\pi_P(x_j)\lambda_P}} \tag{5}$$

Experimental Design and Results

We designed and implemented laboratory experiments based on the two-stage bargaining model, with a pie size of \$10, with an unbalanced initial allocation of \$3 for the proposer and \$7 for the responder. The experiment was conducted by selecting the Political Conflict option for the Veconlab Bargaining Game.[4] This option implements a two-stage dispute model with a wide array of parameter choice possibilities. This is a web-based program, and the instructions are configured automatically to match the setup parameters selected. Each bargaining game began with the first mover proposing a redistribution of the money (higher, lower, or the same as the initial status quo allocation). The second mover ("responder") could either accept the proposal or reject and make a counter offer for dividing the total allocation, which drops by \$1, from \$10 to \$9 in the second stage.). If a counter offer is made, the first mover can either accept or reject and initiate a conflict, where the outcome is determined by known probabilities and conflict costs. In all rounds of our experiments, we kept the conflict costs for each player at \$2. The set of feasible demands and counter-demands was restricted to integer values (0, 1, ... 10).

The proposer win probabilities were set at either $p = 0.2$, 0.4, 0.6, or 0.8, with the outcome explained in terms of a throw of a 10-sided die. Each session consisted of 12 subjects being randomly matched for 10 rounds, with one value of p used in the first 5 rounds, and a switch to another value of p in the final 5 rounds. We ran 8 sessions, with treatment sequences of (.2, .6), (.6, .2), (.2, .8), (.8, .2), (.4, .6), (.6, .4), (.4, .8), and (.8, .4). Thus we ran each

[4] http://veconlab.econ.virginia.edu/bg/bg.php

possible combination with a low win probability followed by a high probability, or vice versa. The 96 subjects were University of Virginia students who were paid $6 for participating, plus half of accumulated earnings. Earnings were in the $18-$25 range, for a one-hour session, and were paid immediately after each session.

The quantal response probabilities of initial proposer demands can be used to calculate the expected value of the initial demand, as a function of the four proposer win probabilities. With a high rationality parameter of $\lambda = \lambda_P = \lambda_R = 10$, the predicted initial demands are 1.00, 3.04, 5.07, and 7.03, which essentially match the Nash predictions of 1, 3, 5, and 7 that are easily obtained with standard backward induction arguments.[5] These Nash predictions are connected by the dashed line on the left side of Figure 1, and the Nash zero-conflict proportions are shown by the dashed line on the horizontal axis of the right side of the figure. Since equilibrium predictions (Nash and QRE) are more likely to be relevant after a period of adjustment, we will compare predictions with "last-half" data taken from rounds 6-10. Maximum likelihood estimates of λ using last-half data are in the range between 1 and 2 for various specifications, and the left side of Figure 1 shows these average demands by treatment, along with QRE predictions for $\lambda = 1.6$.

****Figure 1 About Here****

Notice that the average observed demands in the left panel of Figure 1 are generally higher and to some extent "flatter" than the Nash predictions. This pattern is tracked to some extent by the QRE predictions, although the QRE prediction is clearly too high for the strongly asymmetric treatment with a win probability of 0.8. Proposers are not as aggressive as they "should" be in this treatment, which could be due to strong responder aversion to a

[5] The Nash predictions can be verified directly, under the assumption that a responder will accept an offer that makes the responder indifferent, as assumed in the one-stage bargaining model. For example, if the proposer win probability is 0.8, the proposer's expected gain in the second stage is (0.8)*9 minus the conflict cost of 2, which yields 7.2-2 = 5.2. The responder could avoid the conflict by making the next higher integer offer of 6 to the proposer and keeping the remaining amount, 3, from the second-stage pie. In the first stage, the proposer can therefore, offer the responder 3 and keep 7, which is the Nash first-stage demand, assuming that the responder accepts in the case of indifference. The other Nash predictions can be verified in a similar manner.

large loss relative to the status quo.[6]

A particularly important feature of the data is the substantial number of conflict outcomes, which is not surprising given the asymmetry of the initial status quo allocation and the asymmetric win probabilities in all treatments. The right side of Figure 1 shows the conflict proportions across treatments for rounds 6-10, along with predictions derived from the QRE model with $\lambda = 1.6$. The QRE predictions and observed conflict proportions are in the same general range, which is notable given that the predicted proportion of conflicts for the Nash equilibrium is 0 in all treatments. This is important in the context of war given that nearly all work on international conflict takes as given that states avoid conflict under full information. Another interesting feature of the data is that conflicts are slightly more common when there is an imbalance of power; although the incidence of conflicts is relatively flat as the proposer win probability is increased on the range from a weak position ($p = 0.2$) to a strong position ($p = 0.8$). This is because weak proposers make modest demands and strong proposers make aggressive demands. This also is intriguing with regard to the literature on interstate conflict which largely takes demands in interstate crises to arise exogenously to power.[7] The effect of the win probability on average proposer demands is in the predicted direction for all 8 sessions, a result that is significant using a standard non-parametric test.

Data from the first round of play offer a quite different prespective, as shown in Figure 2, where the rationality parameter used in the QRE predictions has been reduced by an order of magnitude from 1.6 to 0.2. The initial proposer demands are clearly much flatter in the first round data, and conflicts are much more likely when there is an imbalance of power in favor of the challenger, as indicated by the strong upward slope of the conflict proportion

[6]An alternative explanation could be based on "probability weighting," i.e. that proposers under-perceive the 0.8 probability of winning and that responders over-perceive the 0.2 probability of winning in this treatment. Probability weighting, with an inverted "S-shaped" form has been used to explain anomalous behavior in other contexts, e.g. the Allais paradox (Tversky and Kahneman, 1992; Wu and Gonzalez (1999) de Palma et al., 2008) and deviations from Bayesian information processing (Holt and Smith, 2007).

[7]Notable exceptions to this include work by Powell (1999), Werner (1999), Reed, Clark, Nordstrom & Hwang (2006) and Reed, Clark, Nordstrom & Hwang (2008).

line on the right side of Figure 2.[8]

****Figure 2 About Here****

Although high-noise calculations seem to approximate the average outcomes *across treatments* for the first-round data, this is not the whole story, since these calculations overstate the spread in decisions *within* a given treatment. Moreover, a change in the proposer's status quo allocation, say from $3 to $5, has no effect on the predictions in this simple model, but it would likely have an effect in the experiments. As Tversky & Kahneman (1992) have shown, loss aversion relative to a reference point, a status quo for instance, implies a kink in the value function that induces risk aversion for decisions that span both gains and losses, as is the case with our conflict bargaining games.[9] The flatness of proposer offers relative to theoretical predictions, especially in the first round of our experiments, indicates that the natural reference point is the status quo allocation, something we will investigate in future work.

Implications

What light does this model shed on bargaining in world politics? First, the experiments highlight the frequency of conflict under complete information; this raises the question of how or why conflict can arise so readily among fully informed bargainers. Moreover, if conflict abounds in full information in a tightly controlled setting like these experiments, would the same dynamic increase the chances of conflict in the uncontrolled, complex, and uncertain interactions of states considering war? Our expectation, one we are developing in other work, is that bargainers have beliefs about their opponents that shape bargaining behavior, and

[8]The first round results exhibit more noise, a higher proportion of conflicts across treatments, and a strong correlation between the challenger's power and initial demands that is not present in the last-half data. Further experimentation and modeling, with matched single-round and multi-round treatments, are needed to isolate the causes of these differences.

[9]Carlson and Dacey (2004, 2006, 2007) also show loss aversion to produce risk averse rather than risk seeking behavior.

steer bargainers away from standard equilibrium strategies. In world politics, states bargain in the shadow of war (i.e., where war is the outside option), relatively infrequently; states pursue the outside option of war even less frequently. States' leaders have finite tenures in office that further limit experience in bargaining in the shadow of war. These are among the reasons we can anticipate states to pursue bargaining strategies that are not completely 'rational,' and that are shaped by noisy beliefs.

The model and experiments also point to the possibility that processes other than limits to rationality interfere with equilibrium strategies. The status quo itself is probably influential in determining demands to the extent bargainers are driven by aversion to loss. Additionally, if the status quo has value as a focal point, it may exert gravity such that significant departures from the status quo are difficult. Finally, bargainers may value what they see as equity and be drawn either away from equilibrium demands or toward focal points like even distributions. States' leaders may be prone to any of these forces as well, particularly if departures from the status quo imply the possibility of losses for which they pay political penalties at home. The normative arguments about democracy and foreign policy also suggest that notions of fairness and compromise may influence interstate decisions about conflict. Interstate bargaining decisions may likewise be subject to similar beliefs about equity that point toward demands that depart from equilibrium.

The primary goal of this paper was to contribute to the dialogue on bargaining models in world politics by providing the first experimental assessment of the ultimatum bargaining model in the context of international conflict. Using experimental techniques we were able to address some of the serious issues that plague non-experimental efforts to test bargaining hypotheses. By focusing on both the patterns of demands and counter offers and on the incidence of conflict as we vary the win probability across treatments, we were able to assess the key elements of Fearon's original model. Our results reveal several novel findings about power, demands, and conflict.

One implication of bargaining experiments in economics is that negotiations do break down and conflict occurs, even in the case of perfect information. This appears to happen in the context of interstate war as well. Despite the common assumption that full information results in a non-conflict outcome, neither Nash nor QRE equilibria are strong approximations of the pattern of conflict in the experimental data, although the QRE and the experimental data do converge at the extreme values of the proposer's win probability.

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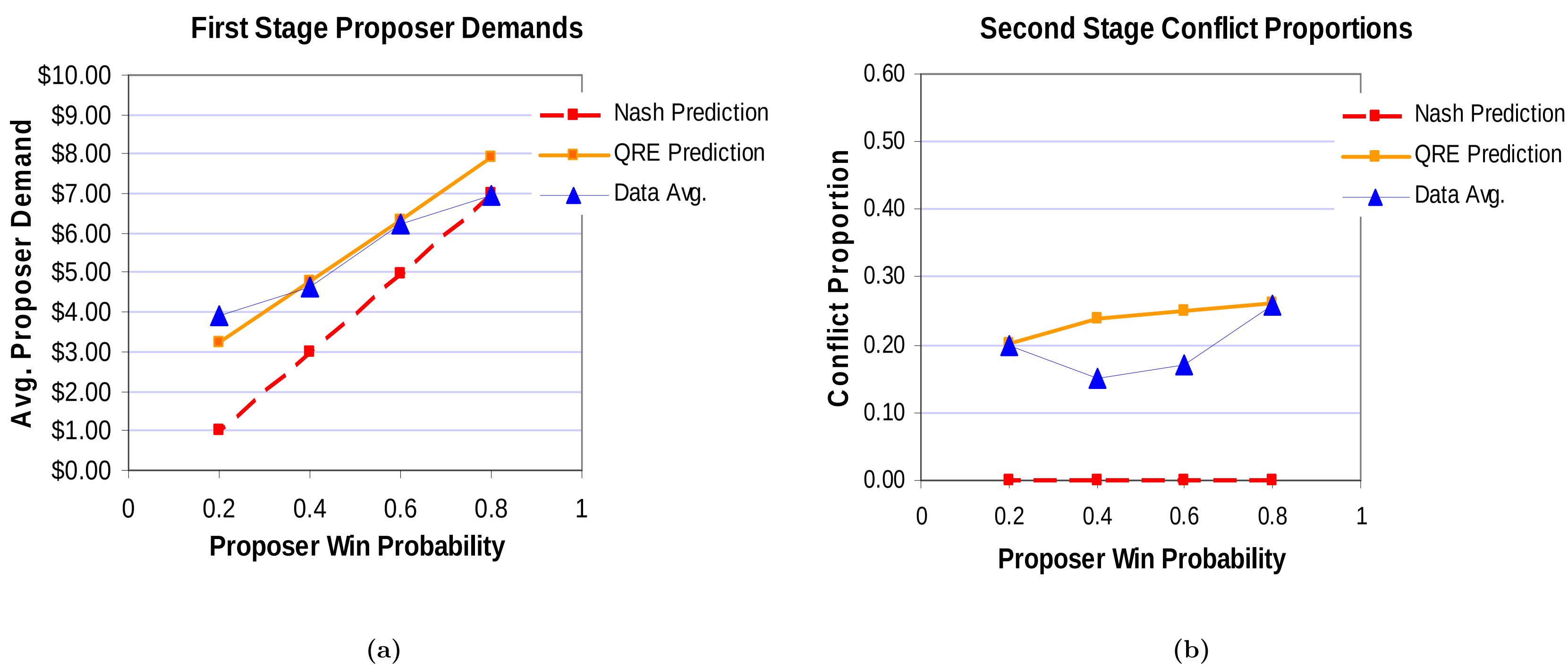

Figure 1: Last Half Data: Observed and Predicted Demands and Conflict Proportions

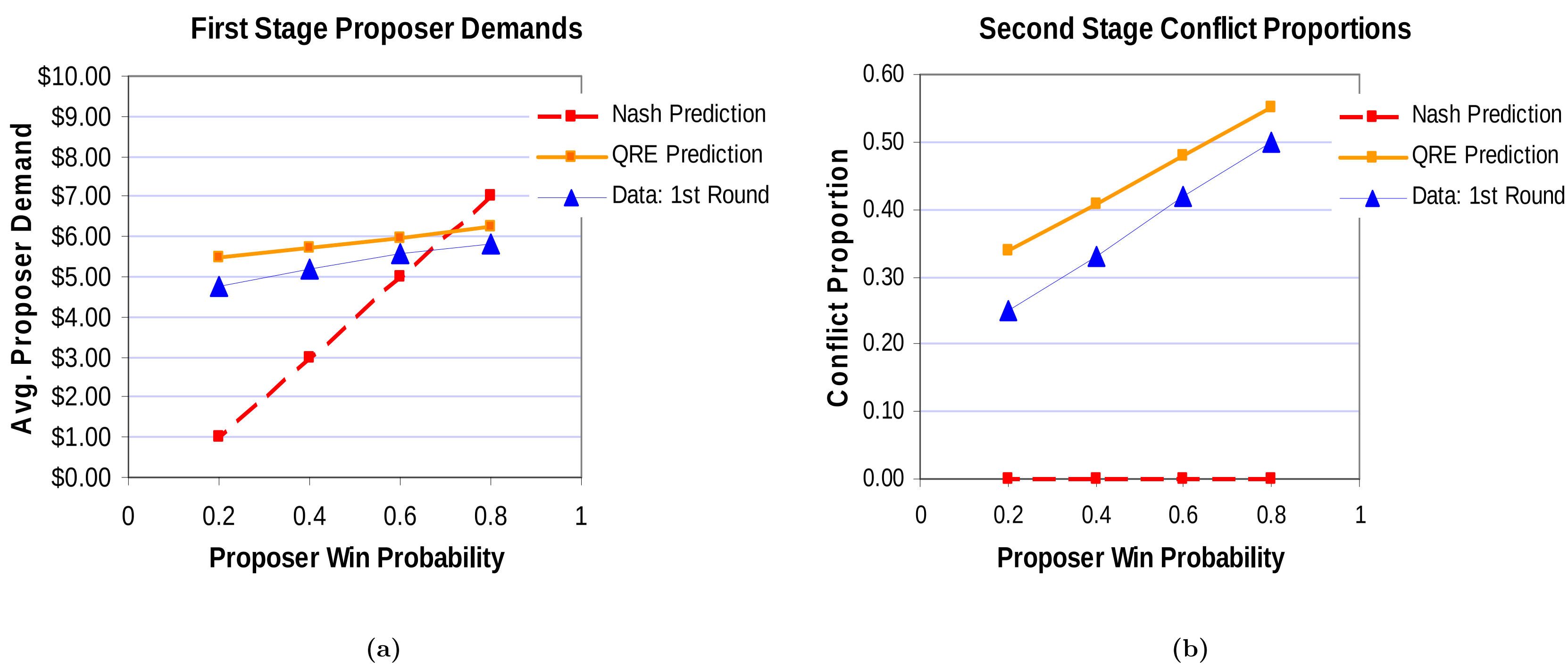

Figure 2: First Round Data: Observed and Predicted Demands and Conflict Proportions