

Assessing the probability of the referendum paradox : the French local election case

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Abstract

In an election, the winner in terms of seats, does not have necessarily a majority of the votes. This phenomenon is known in Social Choice Theory as the referendum paradox. Using data for the French “cantonal” elections from 1985 through 2004, we first try to identify the departments for which such a phenomenon occurs, and to determine the occurrence of this paradox. Then, we compare these results with those of the theoretical models (Feix and al. 2004).

Key words: Referendum paradox, French “Cantonal” election, Probability.

JEL Classification: D7.

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1 Introduction

In democratic systems, the ideal representation should enable each voter to have an equivalent weight, i.e. “one man one vote”. This may mean two very significant things in a system of indirect election. First, the number of elected representatives allocated to each district, shall be proportional to the number of voters enrolled in the electoral register for that electoral district. Secondly, *ceteris paribus*, the party which receives the most votes in the country should control the most seats. The second condition is not always guaranteed. This situation is known in Social Choice Theory as the referendum paradox.

Many studies showed that the emergence of such paradox is not rare. Nurmi (1999) shows that the occurrence of the referendum paradox might have been observed in many democracies in which the electoral systems are based on elections in districts. The paradox can appear, as we will see in our examples, in the first-past-the-post system, and also in the French electoral system. For the American presidential elections, an instance of this paradox occurred in the 2000 election (see table 1).

Table 1: The American presidential election of November 2000.

Candidates	Votes	% Votes	Electors
George W. Bush	50,456,002	47.88%	271
Al Gore	50,999,897	48.38%	266
Ralph Nader	2,882,955	2.74%	0
Pat Buchanan	449,895	0.42%	0
Harry Browne	384,431	0.36%	0
Howard Phillips	98,02	0.09%	0
John Hagelin	83,714	0.08%	0
Autres	51,186	0.05%	0
Total	105,405,100	100%	537

Source : <http://www.fec.gov>

George W. Bush was elected president of the United States by winning more States, 30 States (21 to A. Gore), for a total of 271 electors among 537 in the Electoral College (266 to Gore), whereas he obtained less votes than his competitor, 47.9% to 48.4%. This reversed majority happened four times in the American presidential election history, in 1824, 1876, 1888 and 2000. In the same way, during the 20th century, the winning party in the United Kingdom Parliament, in terms of seats, had less popular support than the losing one (see table 2) in four elections.

France, is not immunized against this phenomenon. This article will be devoted specifically to study and illustrate the referendum paradox using the French cantonal electoral data.

The French cantonal elections are based on elections in districts; they are, consequently, likely to encounter this paradox. Let us briefly remind that the purpose

Table 2: United Kingdom Parliament election.

Year	Parties	Votes	% Vote	Seats
1910	Conservative party	3,104,407	46.8%	272
	Liberal party	2,866,157	43.5%	274
	Others parties	696,836	9.7%	124
1929	Conservative party	8,656,225	38.1%	260
	Labour party	8,370,417	37.1%	287
	Others parties	5 621 733	24.8%	71
1951	Conservative party	13,718,199	48.0%	321
	Labour party	13,948,883	48.8%	295
	Others parties	929,512	3.2%	9
1974	Conservative party	11,872,180	37.9%	297
	Labour party	11,645,616	37.2%	301
	Others parties	7,822,366	24.9%	37

Source : <http://psephos.adam-carr.net>

of these elections is to elect the members of the district council for the departments: The jurisdiction for this election is the canton, in which a councillor is elected. The district council is renewed by half every three years using a run-off system.

The objectives of this paper are twofold: first, using the data for the cantonal elections in metropolitan France from 1985 to 2004, we want to identify the departments for which such a phenomenon of the reversed majority occurs. Secondly, we try to estimate empirically the occurrence of the paradox. We could then compare our results with the theoretical predictions (see Feix et al. (2004)). The interest of the cantonal elections is to present a long series of data, with 93 departments voting every 3 years . Thus, we have 651 (93×7) cases to study.

To aggregate the votes within each department, we employ four approaches: (1) votes cast in favour of the winners, (2) votes cast in favour of all the candidates of the two major political camps, (3) taking into account of the presence of tiers parties and (4) the distinction of the two rounds of elections.

In this study, we give first a detailed exemple of the referendum paradox in the second section. Section 3 motivates four approaches to estimate the referendum paradox in cantonal elections. Section 4 provides our results. Section 5 is devoted to a discussion comparing the four approaches, and compares our results with the theoretical model. The last section concludes the paper.

2 The referendum paradox

2.1 The referendum paradox : illustration

Habitually, important legislative decisions are decided in the parliament, but some particularly crucial decisions are subjected to a referendum. This one is a direct vote

in which an entire electorate is asked to either accept or reject a particular proposal. Such referendum is an usually optimal democratic solution in the representative democracy. Nevertheless, in some countries where the referenda are consultative and not obligatory, a particular problem can emerge: Which one of the two results can be considered more legitimate; the result of the referendum or the parliamentary result of the vote? It may happen that the majority of the voters favour an opinion and the majority of the representatives opt for its negation. To illustrate this phenomenon, we present Nurmi’s exemple (Nurmi 1999). Let us suppose, moreover, the issue in question is the adoption of a new constitution, and this one requires a national referendum. The referendum result is dichotomous: “Yes” or “No”. Assume that a decision of binary type Yes/No must be taken in a country with 62 million register votes. This country is divided into 200 districts, each one composed of 310 000 votes and represented by a unique representative. The jurisdictions are assumed to be of equal size which means that the apportionment is proportional. We display in table 3 the distribution of the opinions on the binary question and the vote of the representatives, which follows the majority opinion of their districts. Then we notice the occurrence of the referendum paradox.

District	$D_1 \dots D_{150}$	$D_{151} \dots D_{200}$
Yes	110 000 . . . 110 000	310 000 . . . 310 000
No	200 000 . . . 200 000	0 . . . 0
MPs	No . . . No	Yes . . . Yes
	150 No	50 Yes

Table 3: The referendum paradox

The new constitution, subjected to the popular referendum, is approved because it obtains

$$150 \times 110000 + 50 \times 310000 = 32000000 \quad \text{Yes}$$

against:

$$150 \times 200000 = 30000000 \quad \text{No}$$

We suppose now that the referendum was only consultative and that the final decision comes to the parliament. The representatives being aware of the opinions of their voters, they would vote according to the majority opinion in their constituency. Then the proposal receives only 50 votes against 150. The decision of the MPS goes in opposition to that of the population.

This phenomenon is inevitable, and it can occur whenever the decision is not taken directly by the people, but by locally elected representatives.

2.2 What is the solution?

Basically, the referendum paradox emphasizes the irreconcilable nature of the direct and indirect vote. Recent works on social choice theory (Laffond and Lainé (1999), Chambers (2003), Perote Péna (2006), Bervoets and Merlin (2005,2007)) showed

that no indirect voting procedure can be translate perfectly the results into direct vote result, for any specification of the preferences. Then, Both indirect and direct voting procedures can lead to opposite results. A way to circumvent there negative conclusions is to study the probability of the paradox.

In social choice theory, the pioneer work of Arrow (1963) caused a natural reaction which consisted in wondering about the real importance of the “impossibility theorems” for the practical one. Many works during the recent years tried to estimate the probability of the different social choice paradoxes, including the Condorcet effect, the manipulation, etc.. One owes a complete panorama on these attempts in Gehrlein (2006).

Concerning the referendum paradox, Feix and al. (2004) studied it with a priori two models of vote. Indeed, they considered a competition between two parties in N districts of equal size. In a first model (Impartial Culture), one supposes that every voter plays heads or tails to determine his vote. Then, when the number of districts increases, the limit of the paradox tends to about 20.5%. With a second model (Impartial Anonymous Culture), a slight correlation between the voters choices inside the same district makes the paradox probability tends to 16.5% when N tends to infinity. These results must be confronted to the reality. An empirical study will allow us to make comparisons between the empirical and theoretical results. To carry out this study, we have to choose a sufficiently rich electoral database. It is the case of the cantonal elections in France.

3 Data and methodology:

3.1 Data:

The cantonal elections allow the population to choose their representatives to district councils of their department (Grigny 2004). Indeed, every department is divided in cantons (on average 39 cantons per department) that designate a councilor by the means of a plurality run-off system. A councilor is elected for six years, but the renewal of the district council is done partly every three years. To be elected in the first round, the candidate must get the absolute majority of the votes cast, as well as a number of votes at least equal to the quarter of the number of the registered voters. To be qualify to the second round, it is necessary to get more than 10% of the registered voters. **If a single candidate fulfils these conditions**, the person who is ranked at second place also goes to the second round. During the second round, the candidate who obtains a plurality of votes is elected.

The data in the present study are extracted from the electoral database realized by the Quetelet center (based upon the ministry of the interior) and the LASMAS (based upon French Statistical institute: INSEE). They are the data of cantonal elections of Metropolitan France, which took place between 1985 and 2004. We excluded from the database Paris, Corsica, and Overseas territories **which either name specific voting role, or present a much diverse spectrum of political forces**. For more detail, we have approximately 1915 cantons per election year, that is

13405 election data over a period of 20 years.

From each election arises a lot of discussions and comments. Nevertheless, the central subject remains the modeling of the political scene and the evolution of left/right divide as well as the respective force of the various components of these two blocks. Many factors can influence the balance of power in the cantonal elections¹. The outcome of this balance, in elections intervened since 1985, is displayed in table 4. The balance of power was rather in favor of the right. The left, during the last 20 year often gathered up then 45% of the vote, *with an improvent*. But it knew a turnaround position since 2001.

Table 4: cantonal election from 1985 (metropolitan France).

		1985	1988	1992	1994	1998	2001	2004
% vote Total, 1 st round	T.Gauche	41.3	47.8	34.4	40.8	44.6	46.4	48.2
	Others [±]	0.8*	0.2*	10.3*	4.4*	1.2	1.8+	1.8+
	T.Right	49.1	45	43	44.8	40.3	44.7	37.5
	FN	8.8	5.4	12.3	9.9	13.9	7.1	12.5
Total seats	T.Left	642	842	547	804	977	1102	1149
	T.Right	1310	1093	1395	1110	977	813	796
General council Presidencies	T.Left	27	27	20	20	33	37	47
	T.Right	68	68	75	75	62	58	48

[±] This category gathers Ecologists, Divers, CPNT and the Greens. * including the greens, 0.8% in 1985, 1.6% in 1988, 8% in 1992 and 2.6% in 1994. They counted in the left camp from 1995. + including CPNT, 0.4 in 2001 and 0.1 in 2004.

sources: adapted from Martin (1998, 2004a) and Le Gall (1998, 2001, 2004).

Certainly, since it is a matter of a local ballot, it is easy to note here and there, in several districts, that the ballot is strongly marked by the personality of incumbent candidates whose influence transcends the partisan divides Independently of their political orientations, their electorate is often wider than the one of their camp.

Of course equally, all the nuances of the political spectrum are not represented in every district, even in first round, and the choice of the voter is often limited and channeled towards the closet candidate to his opinion.

Nevertheless, the votes totalisation between the major political coalitions seem to be delicate on a department scale and depend on the “official” labels of the candidates. Let us note, in this respect, that our study will be based on the classification of the candidates proposed by the government services.

In addition, before specifying methodologies adopted here, it is important to reconsider the objective for which several adjustments, on these data, will be operated. This objective is to evaluate the frequency of the paradox. For that, it is necessary for us to build a pertinent method to measure this occurrence while taking into account the specificities of the cantonal ballot in comparison with the idealized

¹For more detail, one can consult the papers of Lancelot (1985), Parodi(1985), Guhur(1988), Portelli(1992), Habert, Perrineau et Ysmal (1992), Jaffré(1992, 1998), Van Tuong(1992), Le Gall(1994, 2001, 2004), Martin(1994, 1998, 2004a, 2004b), Chiroux(1994), Guastoni(1998) et Grigny (2004).

structure, presented in the table 3. Two obstacles to overcome are to establish an effective measure:

- The existence of two rounds instead of one
- The frequent presence of a third party in the second round

Another difficulty is due in part to the definition of the referendum paradox itself: it is necessary to consider the case where the two political forces obtain the same number of seats (and not the same number of votes) as paradoxical? Now, we retain only two versions of definition of this paradox. In the first case, we consider that all the situations of equality are paradoxical cases. This analysis is then in the logic of the strong version of the paradox. In the second, on the opposite, we do not suppose that all these situations are paradoxical because the party which has more votes must logically have more seats, it is here, in the logic of the weak version.

Another question is how to interpret the definition of the paradox when the total number of seats is odd ($2k + 1$) and a party obtains the integer part of this total number divided by 2, i.e. k . An interpretation latitudinarian considers that this situation does not constitute a paradoxical case, because the popular winner is not beaten with the voting rule. On the other hand, one can regard all these situations as paradoxical, if it is supposed, in theory, that the popular winner should win with no discussion. In this case, one remains within the framework of the weak version. This section is intended to analyze the reversed majority cases where the variation of the gain in terms of seats is equal to at least one seat, except contrary motion.

Then the problematic of votes aggregation can be summarized in the following questions. How do we have to aggregate votes obtained by the different political forces? Is it really necessary that the extreme right coalition must be taken in consideration, knowing that its presence in terms of votes has not been negligible until the last electoral years? Does the separation of the two rounds bring an additional explanation to the treated phenomenon?

3.2 Four methods to estimate the paradox:

To answer the above mentioned questions, we circumscribed our methodology to four approaches.

First approach: votes cast in favor of the elected representatives

Firstly, we adopt a method which consists in measuring the occurrence of the paradox according to the votes obtained by the elected councillors only. More precisely, at the level of every district, we keep only the votes that allowed a candidate to be elected, whatever the round it was (first or second), and consider the votes of the other candidates as null. We aggregate, afterwards, these votes by department and by political camps, to examine if the elected candidates of the majority camp in the council are elected with more ballots than the ones of the minority camp.

Second approach: all votes cast in favor of all the candidates

Next, we consider the phenomenon by taking into account, this time, all expressed votes in favor of every party. In other term, we keep all votes collected by the right and left candidates that were elected or not. In fact, this second approach is based on the following principle: we identify the votes received by every political coalition within every district then we aggregate them at the level of the department. After that, we compare the number of seats and that of votes gathered by every political camp in order to show the existence or the absence of the paradox. Notice that we don't separate between the votes and the seats obtained in both rounds of balloting. We aggregate the result of "decisive ballot", as if the election concern the first-past-the-post system.

Third approach: the presence of a third party

The score of the extreme right is, over the studied period, sometimes considerable. This party constitutes a third block which can rock the result of a poll. It seems judicious to integrate in our analysis the impact of the presence of a third camp, especially in the second round. Our third step proposes to re-examine the calculation of the paradox with respect to classification left-moderate right-extreme right. Besides the distinction of votes obtained by the left and the right, we distinguish votes collected by the moderate right and the extreme right. We recalculate afterwards the occurrence of the paradox while comparing the seats and the votes for each of the three political forces. We adopt the previous principle concerning the votes, i.e. , we incorporate the votes obtained by all the candidates (victorious or not). The objective of this method is to know whether the introduction of an important third political party changes the balance of power.

Fourth approach: the distinction between the rounds

In the previous approaches, we aggregated the electoral data from either the first and the second round, depending on the round where a candidate was declared victorious. Thus, we completely ignored the fact that the participation might rises or declines in the second round, the fact that there are much less candidates at the second stage, etc. . . . A robust assessment should then separate the results of the two rounds, analyzing on one hand the cantons that have been declared after the first round, and on the other hand the cantons that have chosen their winner only at the second stage. Therefore, a fourth and last step is suggested. This method is based on the distinction between the rounds as well as on the aggregation of the votes obtained by every competitor according to the classification (right, left and extreme right). The goal is to provide measures of the paradoxes which are not affected by possible modifications of the voting behavior between the two rounds. Before beginning to present our results, we give some remarks. According to these four way of defining the referendum paradox, we added the votes of a camp as if it were represented by the same candidates in each canton. But, actually, we are aware that we have aggregated very heterogeneous data, and that each of the above mentioned method is a tentative reconstruction of simple two-party-system.

However, we will show that despite all these problems, the existence of a referendum paradox is clearly highlighted for several departments, whatever the applied method.

4 Results : Assessing the referendum paradox

This section is devoted to the evaluation of the referendum paradox with the four approaches. To complete the analysis, we develop the reason for choosing an approach compared to another, providing and explaining concrete examples.

4.1 Estimating the paradox with the votes cast in favour only the elected members

The detailed statistics of the first approach appear in the table 5. The results show that the cantonal elections, since 1985, were far from the absence of the referendum paradox. The department of Aveyron in 2004 is a good example. There are 106 563 registered voters and 23 seats at stake within this department. The right parties succeeded in conquering 14 cantons whereas they collected only 17.64% of the votes of the registered voters. On the other hand, the left winners gathered 9 cantons with 26.43% of the poll.

Table 5: Departments where the paradox appears according to the first approach.

Year	department	Registered voters	Seats at stake	Left		Right	
				% Vote	C	% Vote	C
2004	AVEYRON	106563	23	26.43%	9	17.64%	14
	CALVADOS	202505	23	19.68%	11	16.09%	12
	COTE D'OR	157377	21	19.80%	10	16.31%	11
	JURA	82212	17	18.84%	8	16.23%	9
	SARTHE	202492	19	18.56%	8	16.89%	11
	VIENNE	147229	20	20.89%	9	15.89%	11
2001	ALPES DE HAUTE PROVENCE	50182	14	18.99%	8	19.33%	6
	CHARENTE	135987	17	16.40%	7	15.90%	10
	GARD	243307	23	13.35%	12	21.73%	11
	OISE	259068	21	15.59%	11	15.81%	10
	SAONE ET LOIRE	220949	29	17.11%	13	15.99%	16
	TARN	138956	23	19.26%	12	19.55%	11
1998	CHARENTE	126877	18	15.38%	8	13.98%	10
	CHER	117054	18	15.21%	7	14.73%	11
	CORREZE	92762	19	18.81%	9	18.03%	10
	GERS	64888	17	19.21%	7	18.80%	10
	INDRE ET LOIRE	173372	19	14.93%	8	13.72%	11
	MEURTHE ET MOSELLE	261967	25	15.64%	12	11.57%	13
	PYRENEES ATLANTIQUES	210259	26	17.06%	11	15.10%	15

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... table 5 continued

	RHONE	449295	26	13.35%	11	12.43%	15
1994	AISNE	190775	21	22.06%	10	13.54%	11
	ALLIER	126474	17	15.86%	9	16.52%	8
	ARDECHE	116357	17	17.69%	8	17.21%	9
	CHARENTE MARITIME	188721	25	15.79%	13	17.51%	12
	DOUBS	168749	17	16.95%	8	16.07%	9
	EURE	176768	21	16.32%	8	15.73%	13
	EURE ET LOIR	131241	15	15.32%	6	15.24%	8
	NORD	806162	38	16.70%	18	16.16%	20
	HAUTES PYRENEES	97169	17	18.67%	9	19.01%	8
1992	HAUTE GARONNE	327989	25	13.91%	14	18.17%	11
	LOT	69466	17	17.95%	10	20.83%	7
	HAUTES PYRENEES	75018	17	17.53%	9	18.60%	8
1988	AISNE	188596	21	17.73%	10	13.10%	11
	ARDECHE	111016	17	15.32%	9	16.89%	8
	CHER	107058	17	15.43%	8	15.02%	9
	DORDOGNE	136487	24	19.63%	11	18.64%	13
	SAONE ET LOIRE	217713	29	15.72%	14	12.68%	15
	SOMME	174487	23	16.82%	10	16.81%	13
1985	BOUCHES DU RHONE	478524	23	13.67%	13	15.90%	9
	COTES D'ARMOR	208366	27	20.25%	14	20.57%	13
	CREUSE	49454	13	17.35%	7	21.71%	6
	DROME	141271	19	17.75%	11	18.25%	8
	PUY DE DOME	215373	30	18.89%	16	20.41%	14
	HAUTE SAONE	90479	18	20.05%	8	19.89%	10
	VAUCLUSE	141694	12	18.50%	7	19.98%	5

Although there are less paradoxical cases in 1992, in the other years, the occurrence of the referendum paradox is rather frequent. This paradox appears on average 6.42 times among the 93 departments, for each cantonal election year.

In addition, this table also reveals that the paradoxes does not always occur in the same departments, every six years. In other words, the departments which know this paradoxical phenomenon vary from one electoral year to another. For example, the department of Calvados suffered from the paradox only in 2004, while the department of Aisne was touched by the phenomenon in 1988 and 1994 (same cantons) and the department of Charente encountered the phenomenon in 1998 and 2001, with a different set of cantons at stake.

All in all, the departments coming up, at least only once, against the problem of the reversed majority, account for 41.93% of the departments (39 departments are touched among the 93). Moreover, we count, on the 651 studied cases, 45 paradoxical situations, which means that the frequency of the paradox is 6.91% (45/651). This result seems obvious given the instability of the balance of power between the two major political coalitions, Left-Right, during the period. Another remark which deserves to be mentioned is that the paradoxes does not systematically favor one camp. By way of illustration, in 2004, the right wins more seats with less

vote in all the departments suffering from the paradox, while, in 1985, the left gains the majority of the seats with less vote, in all the paradoxical cases. In fact, when one coalition does badly nationwide during an election year, the strength of the incumbent candidates helps it to hold back more seats than expected in some departments, leading then to a referendum paradox.

Not to take into account the all votes of the parties of the same political coalition clearly underlines a limit of this approach². The example of the department of Aveyron (2004) attests this: The left carried out a score of 60% of the votes - votes only expressed for its elected candidates - against 40% for the right. However, it was trailed by 4 point (48% against 52%) when all the votes cast in favour of the two political forces were taken into accounts. This leads to reexamine the probability of the paradox according to the second approach.

4.2 Second approach: all votes cast in favour of all the candidates

The results displayed in 6 have similarities and divergences compared to the first approach. They confirm the presence of reversed majorities. Moreover, they show that the number of paradoxical cases increased, during every year except for 1994. The cantonal elections of 2004 in which we counted 11 cases instead of 6 can illustrate our remark. However, the list of departments which suffer from the paradox can drastically change when we compare tables 5 and 6 for a given year. Hautes Alpes (2004) illustrate this fact. The right succeeded in conquering only 6 cantons among the fifteen disputed while collecting 52.50% of the votes, whereas the left get control of 9 seats by obtaining only 47.50% of the votes.

²The votes of the other parties which are classified into various were not taken into account in our analysis for simplification.

Table 6: Departments where the paradox occurs according to the second approach.

Year	department	Votes cast	Councillors	Left		Right		Others	
				% Vote	C	% Vote	C	% Vote	C
2004	HAUTES ALPES	34863	15	47.50%	9	52.50%	6	0.00%	0
	CALVADOS	126833	23	50.27%	11	49.54%	12	0.19%	0
	CHARENTE MARITIME	137151	26	48.97%	14	51.03%	12	0.00%	0
	MAINE ET LOIRE	151373	20	47.54%	10	50.75%	9	1.71%	1
	PYRENEES ORIENTALES	105697	16	47.54%	9	50.00%	6	2.47%	1
	RHONE	291443	27	48.07%	15	51.14%	11	0.80%	1
	SARTHE	122986	19	51.52%	8	48.48%	11	0.00%	0
	SEINE ET MARNE	243037	23	49.56%	14	50.44%	9	0.00%	0
	VAUCLUSE	110352	12	43.08%	9	56.92%	3	0.00%	0
	VOSGES	91153	15	43.14%	8	56.73%	7	0.14%	0
	TERRITOIRE DE BELFORT	30940	8	49.11%	6	50.89%	2	0.00%	0
2001	CHARENTE	78030	17	50.26%	7	49.74%	10	0.00%	0
	CORREZE	65035	18	51.31%	7	48.69%	11	0.00%	0
	DOUBS	100318	17	48.75%	9	51.25%	8	0.00%	0
	ISERE	195964	29	50.34%	14	49.66%	15	0.00%	0
	OISE	137360	21	42.97%	11	57.03%	10	0.00%	0
	PYRENEES ORIENTALES	84170	15	49.79%	9	50.21%	6	0.00%	0
	SAONE ET LOIRE	127130	29	51.89%	13	47.29%	16	0.82%	0
	SEINE MARITIME	250447	35	50.50%	14	49.32%	21	0.18%	0
1998	BOUCHES DU RHONE	308149	27	48.48%	24	51.52%	3	0.00%	0
	CHER	63301	18	51.40%	7	44.40%	11	4.20%	0
	DROME	86977	18	49.37%	11	50.63%	7	0.00%	0
	FINISTERE	187002	28	46.87%	17	52.12%	11	1.02%	0
	HERAULT	192643	26	47.00%	21	48.56%	5	4.45%	0
	ILLE ET VILAINE	155439	27	44.32%	14	53.35%	13	2.33%	0
	ISERE	156067	29	49.14%	16	50.72%	13	0.13%	0
	MEURTHE ET MOSELLE	120257	25	53.23%	12	46.30%	13	0.47%	0
	PYRENEES ORIENTALES	87027	16	45.95%	11	52.76%	5	1.29%	0
	SOMME	129535	23	46.45%	13	52.68%	10	0.87%	0
	VAUCLUSE	90584	12	39.94%	9	60.06%	3	0.00%	0
	VOSGES	79501	15	46.07%	8	53.93%	7	0.00%	0
	TERRITOIRE DE BELFORT	25994	8	49.67%	6	50.33%	2	0.00%	0
VAL D'OISE	137406	20	42.19%	11	52.77%	9	5.04%	0	
1994	AISNE	118883	21	53.92%	10	45.74%	11	0.34%	0
	ALLIER	76200	17	47.95%	9	52.05%	8	0.00%	0
	ALPES DE HAUTE PROVENCE	28538	14	52.45%	6	47.55%	8	0.00%	0
	BOUCHES DU RHONE	262479	26	49.09%	18	50.88%	8	0.03%	0
	CHARENTE MARITIME	106624	25	46.57%	13	53.18%	12	0.25%	0
	DORDOGNE	98145	24	49.57%	14	50.43%	10	0.00%	0
	INDRE	60733	13	50.85%	5	49.15%	8	0.00%	0

...Continued on next page...

... table 6 continued

1994	MEURTHE ET MOSELLE	129529	20	44.20%	12	55.80%	8	0.00%	0
1992	BOUCHES DU RHONE	325881	29	39.87%	18	59.16%	11	0.97%	0
	HAUTE GARONNE	202248	25	48.00%	14	48.03%	11	3.97%	0
	HERAULT	179887	26	43.91%	16	55.87%	10	0.21%	0
	VAL DE MARNE	184996	25	41.19%	16	55.85%	9	2.96%	0
1988	AISNE	98613	21	56.28%	10	43.20%	11	0.52%	0
	ALLIER	69971	17	53.18%	8	46.82%	9	0.00%	0
	ARDECHE	64366	17	49.11%	9	50.89%	8	0.00%	0
	CHARENTE	66037	17	52.11%	8	47.89%	9	0.00%	0
	CHARENTE MARITIME	90309	25	51.48%	12	48.52%	13	0.00%	0
	DORDOGNE	93046	24	51.21%	11	48.79%	13	0.00%	0
	FINISTERE	163890	26	48.48%	15	51.12%	11	0.40%	0
	ISERE	155526	29	49.03%	15	50.76%	14	0.21%	0
	OISE	116975	21	50.04%	10	49.96%	11	0.00%	0
	SEINE MARITIME	195827	35	52.27%	14	47.65%	21	0.08%	0
	SOMME	100203	23	50.39%	10	49.61%	13	0.00%	0
ESSONNE	143794	21	50.54%	6	48.82%	15	0.64%	0	
1985	BOUCHES DU RHONE	287234	23	43.51%	13	56.49%	10	0.00%	0
	COTES D'ARMOR	160237	27	49.48%	14	50.52%	13	0.00%	0
	DROME	94699	19	48.81%	11	51.19%	8	0.00%	0
	HERAULT	170314	23	48.37%	14	51.53%	9	0.10%	0
	LANDES	82362	15	49.48%	10	50.52%	5	0.00%	0
	NORD	553200	40	49.76%	22	49.77%	18	0.47%	0
	PUY DE DOME	152349	30	46.35%	16	53.65%	14	0.00%	0
	VAUCLUSE	97370	12	48.17%	7	51.83%	5	0.00%	0
	TERRITOIRE DE BELFORT	30315	8	49.09%	5	50.91%	3	0.00%	0
	SEINE SAINT-DENIS	189631	20	49.90%	11	50.10%	9	0.00%	0
VAL DE MARNE	251690	30	46.61%	17	52.20%	13	1.19%	0	

C : Councillors.

In general, we have 68 cases out of 651, i.e., the occurrence of the referendum paradox is 10.45%. On annual average, the paradox appears 9.71 times. We can, moreover, affirm that in all of the 93 departments, 41 encountered the reversed majority at least once.

The remarks advanced in the preceding analysis remain valid. In other words, the departments suffering from the paradox differ from one electoral year to another, and the paradox usually favors block which is losing nationwide, thanks to its incumbent candidates.

It should be also noted that neither in terms of seats nor in terms of vote, no party dominates the composition of the general councils continuously: sometimes the right controls more seats without having the majority of the votes and thus the left collects more votes without controlling more seats; sometimes it is the reverse. We quote by way of illustration the department of Charente Maritime. In 1988, the left gleaned 13 seats out of 25 with 48.52% against 12 seats for the right. On

the other hand, in 2004 the left wins the election by receiving 14 seats out of the 26 disputed with 48.97% of the votes. But, in general, the left controls more seats without having the majority of the votes, in 44 listed cases among the 68.

A fact that revealed in table 6 is the strength of the extreme right in 1998 and 2004, and its capacity to maintain many candidates in the run-off these years. However, we note that the departments, which know a strong presentation of this political block, encounter paradoxical cases. We find such examples with the departments of the Pyrénées Orientales (2004) and Vaucluse (2004). The left controlled in the first department 9 seats out of 16 with 47.54% of the votes against 6 seats for the right with 50%, including 8% for the extreme right candidates. With regard to Vaucluse, the left obtained 43.08% of the votes against 56.92% for the right including 25% of the votes of the partisans of the extreme right.

Admittedly, other departments know also strong presence of the extreme right vote and do not encounter a reversed majority. It is the case, in 1998, for Indre-et-loire, Oise, Rhône and Seine-et-Marne. But, by taking into account only the votes of the moderate right a paradox appears in these departments. This means that considering the right extreme as a separate block modify all the analysis, at least in 1998 and 2001. Thus, we need to turn to a third method which labels the right extreme votes separately.

4.3 Third approach: the effect of a third block

Before developing in detail the results, we announce the principal changes arrived in the political formation of extreme right-hand side. Before analyzing in detail the results, we have to describe the principal changes concerning the extreme right party, mainly the Front national (FN), in the last decades. By clinching 11% votes in the European elections in 1984, the FN list created a surprise -between 1965 and 1984 the extreme right always stagnates in less than 1% of the votes. Henceforth, the FN made the decision to present at cantonal elections the candidates in a large number of cantons, modifying, on the right, the range of the choices. Moreover, by regularly exceeding the 10% **there**, hold to go for the run-off, many classical left-right confrontation in the second round **evolved** toward three-way races. With its strategies, the FN succeeded to secure the loyalty of a stable electorate, sociologically and geographically heterogeneous electorate. It ended the series of elections which were marked by an affirmed bipolarization, where each candidate was located clearly in the left camp or the right. Consequently, the influence of the extreme right is felt more and more as a third French political force, starting from the elections of 1984. In this framework, notice especially the performance of FN in the presidential election of 2002 when it was occurred in the second round.

Thus, in our calculations, we will try to answer the following question: Does the introduction of a third political force into the analysis change indeed the results?

Table 7: Departments where the paradox appears according to the third approach.

Year	department	Votes	C	Left		Right		EXR	
				% Vote	C	% Vote	C	% Vote	C
2004	HAUTES ALPES	34863	15	47.50%	9	48.89%	6	3.61%	0
	CALVADOS	126833	23	50.27%	11	46.47%	12	3.06%	0
	COTE D'OR	100635	21	48.72%	10	48.24%	11	2.86%	0
	INDRE	53227	13	46.12%	4	42.56%	8	4.03%	0
	LOIRET	118759	20	41.81%	6	41.19%	12	9.82%	0
	MAINE ET LOIRE	151373	20	47.54%	10	48.19%	9	2.57%	0
	MARNE	108668	22	47.38%	9	43.65%	13	8.68%	0
	SARTHE	122986	19	51.52%	8	47.32%	11	1.16%	0
	VOSGES	91153	15	43.14%	8	48.52%	7	8.20%	0
	YONNE	78956	22	46.14%	8	45.08%	14	8.50%	0
2001	ARDENNES	64839	19	48.71%	6	48.55	13	2.58	0
	CHARENTE	78030	17	50.26%	7	48.30%	10	1.44	0
	CORREZE	65035	18	51.31%	7	47.97%	11	0.72	0
	DOUBS	100318	17	48.75%	9	49.39%	8	1.86	0
	FINISTERE	196763	26	49.34%	11	49.29%	15	1.08	0
	ISERE	195964	29	50.34%	14	47.58%	15	2.08	0
	NORD	424584	39	48.94%	17	44.78%	22	6.13	0
	OISE	137360	21	42.97%	11	54.09%	10	2.93	0
	SAONE ET LOIRE	127130	29	51.89%	13	43.33%	16	3.96	0
	SEINE MARITIME	250447	35	50.50%	14	45.17%	21	4.15	0
	VAUCLUSE	113092	12	48.31%	4	44.31%	8	7.29	0
1998	CHER	63301	18	51.40%	7	43.68%	11	0.72%	0
	FINISTERE	187002	28	46.87%	17	50.39%	11	1.72%	0
	ILLE ET VILAINE	155439	27	44.32%	14	51.95%	13	1.40%	0
	INDRE ET LOIRE	88619	19	49.74%	8	48.58%	11	1.48%	0
	MEURTHE ET MOSELLE	120257	25	53.23%	12	43.76%	13	2.55%	0
	OISE	132213	20	41.59%	8	40.81%	11	16.16%	1
	RHONE	216403	26	42.33%	11	37.53%	15	17.93%	0
	SEINE ET MARNE	178461	23	45.59%	10	44.80%	13	9.61%	0
	SOMME	129535	23	46.45%	13	49.41%	10	3.27%	0
	VOSGES	79501	15	46.07%	8	51.39%	7	2.54%	0
1994	AISNE	118883	21	53.92%	10	41.49%	11	4.25%	0
	ALLIER	76200	17	47.95%	9	50.96%	8	1.09%	0
	ALPES DE H.P	28538	14	52.45%	6	46.18%	8	1.37%	0
	CHARENTE MARITIME	106624	25	46.57%	13	51.34%	12	1.83%	0
	EURE ET LOIR	75760	15	45.64%	6	43.44%	8	10.91%	1
	INDRE	60733	13	50.85%	5	46.37%	8	2.79%	0
	ISERE	175800	29	48.86%	12	48.13%	17	1.71%	0
	MEURTHE ET MOSELLE	129529	20	44.20%	12	55.80%	8	0.00%	0
	NORD	452093	38	47.60%	18	46.66%	20	5.13%	0

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... table 7 continued

1992	HERAULT	179887	26	43.91%	16	44.64%	10	11.23%	0
	TARN ET GARONNE	51707	15	48.30%	6	45.37%	9	4.62%	0
	VAL DE MARNE	184996	25	41.19%	16	47.11%	9	8.74%	0
1988	AISNE	98613	21	56.28%	10	41.48%	11	1.73%	0
	ALLIER	69971	17	53.18%	8	46.64%	9	0.18%	0
	ARDECHE	64366	17	49.11%	9	49.64%	8	1.25%	0
	CHARENTE	66037	17	52.11%	8	47.89%	9	0.00%	0
	CHARENTE MARITIME	90309	25	51.48%	12	47.33%	13	1.19%	0
	DORDOGNE	93046	24	51.21%	11	48.31%	13	0.48%	0
	FINISTERE	163890	26	48.48%	15	50.11%	11	1.00%	0
	ISERE	155526	29	49.03%	15	49.93%	14	0.83%	0
	OISE	116975	21	50.04%	10	47.88%	11	2.07%	0
	PYRENEES ORIENTALES	66127	16	47.17%	5	46.86%	10	4.19%	0
	SARTHE	88796	21	49.69%	8	49.23%	13	1.08%	0
	SAVOIE	52534	18	49.78%	7	47.73%	11	2.22%	0
	SEINE MARITIME	195827	35	52.27%	14	46.46%	21	1.19%	0
	SOMME	100203	23	50.39%	10	48.53%	13	1.08%	0
ESSONNE	143794	21	50.54%	6	48.34%	15	0.48%	0	
1985	COTES D'ARMOR	160237	27	49.48%	14	49.54%	13	0.98%	0
	HERAULT	170314	23	48.37%	14	48.75%	9	2.78%	0
	PUY DE DOME	152349	30	46.35%	16	50.82%	14	2.84%	0
	TERRITOIRE DE BELFORT	30315	8	49.09%	5	50.91%	3	0.00%	0
	SEINE SAINT-DENIS	189631	20	49.90%	11	50.07%	9	0.03%	0
	VAL DE MARNE	251690	30	46.61%	17	47.08%	13	5.12%	0

C : Councillors.

Table 7 provides overall the distribution of the votes and the councillors between the three political forces in the departments suffering from the reversed majority. For simplification purposes, we also do not treat, here, the political parties other than the right, the left and the extreme right. The political scene of Marne (2004) is an example showing a paradoxical situation according to this approach. The moderate right enjoys a majority of 13 seats out of 22 against 9 for the left knowing that it obtained only 43.65% of the votes against 47.38% for the left. As for the extreme right, it did not succeed in transferring the 8.68% from the votes which it collected to a seats.

The comparison with former methods provides some different results. Firstly, the paradox emerges in certain departments while it disappears else where, the departments of Yonne (2004) and Loiret (2004) are proof. Indeed, the former suffering from the referendum paradox according to the third approach and not the second. Therefore, Loiret suffered from the paradox according to the second and not the third approach. Secondly, the number of paradoxical cases slightly fell. On annual average, the paradox emerges in all 93 departments of the metropolis, 9.14

times. Moreover, we count on the whole 64 cases instead of 68, with 45 departments concerned with the phenomenon. Consequently, the frequency of the paradox is 9.83% with 48.39% of the sample suffering, at least only once, from the paradox.

This analysis emphasises the influence of the Front National in certain departments: Bouches du Rhône suffered from the paradox during four cantonal elections and Pyrenees Orientales during three polls, but the result obtained will be different if one opposes the left to the moderate right. In other words, in these departments, the sum of the votes of the right was amplified by that of the extreme right without making it possible to win more seats. Indeed, the peril of the right resides, particularly, in the level of the FN's votes and its capacity with being present massively at the second round on a significant level. For instance, in Bouches-du-Rhône (1985), the FN gathered 13.20% of the votes cast (43.51% for the left and 43.29% for the moderate right) and it was present at the second round in 17 cantons out of the 21 renewable ones, but it gained only one seat (against 13 seats for the left and 9 for the moderate right) (see table 8).

Table 8: The result of the renewable cantons in the department of Bouches du Rhone in 1985.

Cantons	Vote cast	Left	Right	FN	S	E
2	17077	40.72%	40.57%	18.71%	59.28%	L
4	10107	54.63%	45.37%	0.00%	45.37%	L
6	15291	49.07%	50.89%	0.04%	50.93%	R
8	24276	35.50%	32.62%	31.88%	64.50%	L
9	7142	56.58%	42.47%	0.95%	43.42%	L
12	8574	53.42%	46.13%	0.45%	46.58%	L
14	11772	0.00%	48.67%	51.33%	100.00%	FN
18	10327	35.53%	64.33%	0.15%	64.47%	R
19	12102	34.35%	65.63%	0.02%	65.65%	R
21	8351	52.17%	0.00%	47.83%	47.83%	L
22	12456	37.33%	36.58%	26.08%	62.67%	L
23	14476	45.51%	54.41%	0.08%	54.49%	R
31	7261	47.91%	52.09%	0.00%	52.09%	R
33	7347	46.39%	44.98%	8.63%	53.61%	L
34	9429	53.37%	46.10%	0.53%	46.63%	L
35	12333	45.59%	53.89%	0.52%	54.41%	R
36	29631	49.12%	50.88%	0.00%	50.88%	R
41	17502	53.77%	0.00%	46.23%	46.23%	L
43	17272	46.78%	53.20%	0.02%	53.22%	R
45	8427	43.93%	22.69%	33.38%	56.07%	L
46	16200	31.15%	68.78%	0.06%	68.85%	R
Total	287234	43.51%	43.29%	13.20%	56.49%	-

FN : Front national, S : The sum of the votes of the right and extreme right, E : Councillor label.

It is obvious that in cantons 2, 8, 22, 33 and 45, the three-way races (moderate right-left- right extreme), amplified the defeat of the right. The total right largely exceeded the left in terms of votes, however, it is the latter which conquered these cantons.

Table 9: The result of the renewable cantons in the department of Vaucluse in 2001.

Cantons	Vote cast	Left	Right	EXR	S	Others	E	R
2 AVIGNON-NORD	11040	39.53%	60.47%	0.00%	60.47%	0.00%	R	2
6 BOLLENE	11226	65.80%	34.20%	0.00%	34.20%	0.00%	L	2
10 CARPENTRAS-SUD	14899	45.26%	54.74%	0.00%	54.74%	0.00%	R	2
11 CAVAILLON	14043	45.18%	54.82%	0.00%	54.82%	0.00%	R	2
12 GORDES	3501	39.85%	51.21%	6.11%	57.33%	2.83%	R	1
17 ORANGE-OUEST	9623	0.00%	50.02%	49.98%	100.00%	0.00%	R	2
18 PERNES-LES-FONTAINES	6854	43.14%	56.86%	0.00%	56.86%	0.00%	R	2
19 PERTUIS	20268	71.62%	16.45%	11.93%	28.38%	0.00%	L	1

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... table 9 continued

20 SAULT	1529	69.52%	26.23%	4.25%	30.48%	0.00%	L	1
21 VAISON-LA-ROMAINE	6591	70.76%	18.04%	11.20%	29.24%	0.00%	L	1
22 VALREAS	4904	42.15%	57.85%	0.00%	57.85%	0.00%	R	2
24 AVIGNON-OUEST	8614	36.45%	63.55%	0.00%	63.55%	0.00%	R	2
Total	113092	48.31%	44.31%	7.29%	51.60%	0.09%	-	-

R : decisive round.

Let us treat another example (see table 9), Vaucluse (2001): the left mobilized 48.31% of the votes for only 4 seats against 8 seats for the right, obtained with a percentage of 44.31% of the votes. If one includes the extreme right, which obtained 7.29% of the votes, the phenomenon of the reversed majority artificially disappears. The right extreme won apparently over a part of the electorate of the right. However, a meticulous reading of the results of the two rounds separately shows that the presence of the right extreme does not the principal cause of the phenomenon. In fact, the left obtained respectively 67.85% and 40.64% of the votes (with 3 seats and 1 seat) in the first and second round, and the right parties (including extreme right) collected respectively 3.84% and 59.36% of the votes (1 seat and 7 seats). This results shows that the paradox is absent. Even if the score of the extreme right (respectively 10.77% and 5.92% in the first and second round) is included in the calculation of the paradox, the situation was not changed. On the other hand, the result of adding the two rounds revealed the paradox. Can one talk legitimately about the relevance of a paradox when it comes from the aggregation of data of two different rounds when no round not show it? Another legitimate reaction relates to the change of the electoral behaviors which acts especially on the variation of the turnout, and on the electoral behavior of partisan **alienation** candidates during the first round. These facts lead us to remake all calculations by separating the two rounds in order to refine our analysis. The results are exposed in tables 10 and 11.

Table 10: Departments where the paradox happens for seats called at the first round.

Year	department	Votes	c	Left		Right		EXR	
				% Vote	C	% Vote	C	% Vote	C
2004	AIN	20851	5	45.30%	2	40.61%	3	14.09%	0
	ARDECHE	9997	5	47.58%	2	45.65%	3	6.50%	0
	CHARENTE MARITIME	32432	6	45.24%	2	44.70%	4	10.06%	0
	GERS	8660	3	47.49%	1	43.43%	2	6.54%	0
	ILLE ET VILAINE	43628	7	52.76%	3	37.54%	4	5.78%	0
	LOIRET	10890	3	40.73%	1	39.62%	2	19.65%	0
	MAINE ET LOIRE	42465	6	48.54%	2	36.23%	4	9.15%	0
	DEUX SEVRES	32557	7	51.05%	3	41.16%	4	7.79%	0
	ALPES DE HAUTE PROVENCE	8542	5	47.60%	3	48.35%	2	4.05%	0
	CHARENTE MARITIME	46888	9	49.63%	4	42.41%	5	7.96%	0

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... table 10 continued

	EURE	45715	8	53.61%	3	31.01%	5	14.24%	0
	FINISTERE	52307	8	51.85%	3	42.99%	5	4.05%	0
	GIRONDE	72399	11	51.20%	5	36.20%	6	10.56%	0
	NORD	133595	11	45.88%	4	44.27%	7	9.34%	0
	SEINE MARITIME	110284	15	47.98%	5	42.19%	10	9.42%	0
1998	ALLIER	19450	5	51.57%	2	33.13%	3	9.13%	0
	CHARENTE MARITIME	34612	7	44.35%	3	42.41%	4	10.40%	0
	DORDOGNE	11772	5	56.17%	2	38.91%	3	4.92%	0
	INDRE ET LOIRE	14498	4	50.19%	1	39.52%	3	9.02%	0
	LOT ET GARONNE	9137	3	50.04%	1	32.33%	2	11.75%	0
	NIEVRE	14981	6	54.54%	2	33.64%	4	10.35%	0
1994	CREUSE	7654	4	47.97%	1	47.03%	3	3.97%	0
	GERS	13723	5	47.58%	2	47.08%	3	4.78%	0
	GIRONDE	46470	7	49.62%	2	38.79%	5	6.77%	0
1992	DORDOGNE	29291	9	48.42%	4	44.67%	5	4.84%	0
1988	AISNE	44368	11	55.45%	5	39.55%	6	3.84%	0
	ALLIER	11216	3	54.22%	1	44.63%	2	1.15%	0
	CHARENTE	32782	9	50.89%	4	49.11%	5	0.00%	0
	COTE D'OR	18333	8	50.06%	3	47.57%	5	2.37%	0
	MEURTHE ET MOSELLE	12576	3	48.45%	1	45.11%	2	6.44%	0
	HAUTE SAONE	35666	12	50.09%	5	48.17%	7	1.74%	0
	SAVOIE	31442	11	53.57%	5	42.27%	6	3.70%	0
1985	GERS	12141	5	52.48%	2	47.52%	3	0.00%	0
	NIEVRE	16285	5	49.28%	1	49.11%	4	1.61%	0

C : Councillors.

Table 11: Departments where the paradox occurs for seat called at the second round.

Year	department	Vote	c	Left		Right		EXR	
				% Vote	C	% Vote	C	% Vote	C
2004	AVEYRON	54367	14	50.07%	6	49.93%	8	0.00%	0
	CANTAL	10378	3	45.81%	2	54.19%	1	0.00%	0
	LOIRET	107869	15	41.92%	5	41.35%	10	8.82%	0
	MAINE ET LOIRE	108908	13	47.15%	8	52.85%	5	0.00%	0
	VOSGES	62019	8	41.99%	5	48.92%	3	9.09%	0
	YONNE	69623	18	47.58%	7	44.51%	11	7.60%	0
2001	ARDENNES	48123	14	54.33%	6	45.67%	8	0.00%	0
	BOUCHES DU RHONE	245176	21	51.94%	10	45.21%	11	2.84%	0
	CHARENTE	59614	12	51.95%	5	48.05%	7	0.00%	0
	CORREZE	48717	13	53.64%	6	46.36%	7	0.00%	0
	GARD	140061	20	49.26%	9	44.19%	11	6.55%	0
	LOIR ET CHER	40818	8	41.28%	4	48.11%	0	2.61%	0
	MAINE ET LOIRE	56865	9	48.83%	5	51.17%	4	0.00%	0
	MARNE	40212	8	42.53%	5	55.24%	3	2.23%	0
	NORD	290989	28	50.34%	13	45.01%	15	4.65%	0
	OISE	125507	19	44.39%	11	53.37%	8	2.24%	0
	SEINE MARITIME	140163	20	52.49%	9	47.51%	11	0.00%	0
TARN	59543	13	51.71%	4	48.29%	9	0.00%	0	
1998	CHER	58701	17	52.36%	7	43.12%	10	0.00%	0
	EURE	69778	14	43.45%	8	44.45%	6	10.12%	0
	FINISTERE	140788	20	48.31%	14	51.69%	6	0.00%	0
	ILLE ET VILAINE	120552	20	47.37%	13	50.38%	7	0.00%	0
	INDRE	30574	7	53.23%	3	46.77%	4	0.00%	0
	MAINE ET LOIRE	76606	12	46.32%	7	53.68%	5	0.00%	0
	MEURTHE ET MOSELLE	110430	23	51.96%	10	45.92%	13	1.61%	0
	SEINE MARITIME	159181	25	56.26%	12	41.36%	13	2.37%	0
	SEINE ET MARNE	173953	22	46.15%	10	44.50%	12	9.35%	0
	TARN ET GARONNE	36864	11	52.51%	4	45.32%	7	2.17%	0
	VOSGES	73148	13	47.27%	7	50.78%	6	1.95%	0
YONNE	42700	14	45.52%	4	42.11%	9	8.15%	0	
1994	AISNE	67574	12	50.44%	4	49.56%	8	0.00%	0
	ALLIER	59860	13	48.38%	8	51.62%	5	0.00%	0
	ALPES DE HAUTE PROVENCE	19956	9	55.86%	4	44.14%	5	0.00%	0
	ARDECHE	52323	12	49.31%	7	50.69%	5	0.00%	0
	CHARENTE MARITIME	77736	18	47.43%	10	52.57%	8	0.00%	0
	DOUBS	73976	13	49.17%	7	50.83%	6	0.00%	0
	JURA	56537	15	40.77%	8	51.50%	6	0.00%	0
	MEURTHE ET MOSELLE	129529	20	44.20%	12	55.80%	8	0.00%	0
	OISE	122963	16	45.47%	9	47.69%	7	6.84%	0
	VIENNE	47840	10	49.27%	8	50.73%	2	0.00%	0

...Continued on next page...

... table 11 continued

1992	HERAULT	176548	24	43.37%	14	45.34%	10	11.29%	0
	LOIR ET CHER	37857	8	49.11%	5	50.89%	3	0.00%	0
	HAUTE SAONE	40334	11	48.16%	4	47.96%	7	1.27%	0
	SEINE MARITIME	173340	25	50.45%	11	47.19%	14	1.21%	0
	VAL DE MARNE	137299	20	41.03%	13	51.37%	7	7.59%	0
1988	8 ARDENNES	35897	13	53.45%	6	46.55%	7	0.00%	0
	AVEYRON	11995	4	49.24%	3	50.76%	1	0.00%	0
	GERS	26767	8	45.65%	5	54.35%	3	0.00%	0
	LOIRE	91604	15	50.71%	7	49.29%	8	0.00%	0
	MANCHE	47220	11	48.48%	4	45.82%	7	0.00%	0
	MEURTHE ET MOSELLE	92137	17	49.41%	9	50.59%	8	0.00%	0
	MEUSE	20146	7	49.43%	2	47.28%	5	0.00%	0
	NIEVRE	27090	10	59.93%	4	40.07%	6	0.00%	0
	SAONE ET LOIRE	67849	15	49.50%	10	50.50%	5	0.00%	0
	ESSONNE	123418	18	52.84%	6	47.16%	12	0.00%	0
1995	BOUCHES DU RHONE	277353	21	42.90%	11	44.11%	9	12.99%	1
	CORREZE	28863	7	49.01%	4	50.99%	3	0.00%	0
	EURE ET LOIR	65738	10	48.87%	6	51.13%	4	0.00%	0
	HERAULT	167100	21	47.90%	12	49.37%	9	2.73%	0
	SEINE MARITIME	149386	21	53.20%	10	46.80%	11	0.00%	0
	TERRITOIRE DE BELFORT	30315	8	49.09%	5	50.91%	3	0.00%	0
	SEINE SAINT-DENIS	189631	20	49.90%	11	50.07%	9	0.03%	0
	VAL DE MARNE	144917	18	49.40%	11	50.60%	7	0.00%	0

C : Councillors.

4.4 Fourth approach : the separation of the two rounds

As usual, the occurrence of the reversed majority change from one year to another. Moreover, there are departments which know the phenomenon only with the first round like Ain (2004), or only with the second round, Aveyron (2004). In our database, if the paradox appears in the first round and also in the second round, it will not appear inevitably in the two rounds together (Meurthe-et-Moselle in 1988).

By adopting this fourth approach, the number of paradoxical situations decreases, since we enumerate only 34 and 63 cases respectively in the first and the second round. Consequently, the frequency of the paradox is 5.22% in the first round and 9.68% in the second. However, the number of the departments having the paradox, at least once at the second round, increased to 50%.

After the presentation and the analysis of these four approaches, it appears more relevant to bring together the results of these methods. Also, it appears more judicious to make a more detailed comparison with the theoretical model. Thus, we present in the two following sections a comparative assessment.

5 Discussion

5.1 An analytical Comparison of the approaches

The comparative reading of the results with the four ways of measuring the referendum paradox delivers the following remarks. First, according to the four measurement, we obtain 7 departments which present robust paradoxical cases: Charente (2001), Oise (2001), Cher(2001), Meurthe-et-Moselle (1998), Aisne (1994), Allier (1994), Charente-Maritime (1994).

Second, whatever the adopted approach, the data confirm the presence of the reversed majority, but the frequency varies. The highest frequency is given by the second approach. On the other hand, the probability of occurrence of the paradox is lower when the round is analyzed separately. Sometimes, all the seats of a department are called only in the second round. So the first round can not reflect the relation votes/seats. On the other side, we notice that the highest percentage of the departments suffering from the paradox, at least once, occurs with the second round according to the fourth method (46/93). In fact, we concentrate here on the most competitive seats, with less influence of the incumbents.

Moreover, according to the four methods and out of the 93 departments studied for the period, we counted 72 departments where the paradox appeared, at least one year with at least one method, i.e. 77.42% of the departments. In the 21 departments free from the phenomenon, the majority is ensured in terms of seats and vote for one camp each year. These departments all are, indeed, always favorable to a political camp except for that of the Loire Atlantique (see table 12).

Table 12: The 21 departments where the paradox is absent

departments	F ou S	departments	F ou S
6 ALPES MARITIMES	R	61 ORNE	R
9 ARIEGE	L	62 PAS DE CALAIS	L
10 AUBE	R	67 BAS RHIN	R
11 AUDE	L	68 HAUT RHIN	R
43 HAUTE LOIRE	R	74 HAUTE SAVOIE	R
44 LOIRE ATLANTIQUE	S	78 YVELINES	R
48 LOZERE	R	83 VAR	R
52 HAUTE MARNE	R	85 VENDEE	R
53 MAYENNE	R	87 HAUTE VIENNE	L
56 MORBIHAN	R	92 HAUTS DE SEINE	R
57 MOSELLE	R		

F ou S : always favorable to a camp (left or right) or swing

Other departments (23 departments) always favour a political family but suffering from the paradox (see table 13). The paradox in these departments can be explained by the simple fact that fide in terms of vote for one camp is counterbalanced by the resistance of the other camp. For example, the department of Calvados

in 2004 where there was had a tilt to the left in terms of vote with a resistance on the right in terms of seats.

Table 13: The 23 departments which are always favorable to a camp in terms of seats, and where the paradox appears.

departments	Favorable	departments	Favorable
1 AIN	R	41 LOIR ET CHER	R
8 ARDENNES	R	45 LOIRET	R
12 AVEYRON	R	46 LOT	L
13 BOUCHES DU RHONE	L	50 MANCHE	R
14 CALVADOS	R	51 MARNE	R
15 CANTAL	R	55 MEUSE	R
21 COTE D'OR	R	65 HAUTES PYRENEES	L
30 GARD	L	72 SARTHE	R
31 HAUTE GARONNE	L	73 SAVOIE	R
34 HERAULT	L	86 VIENNE	R
39 JURA	R	89 YONNE	R
40 LANDES	L		

From tables 12 and 13 , 11 departments always give the preference to the left in terms of seats including only 4 not suffering from the paradox. However, the number of the departments always supporting the right is more important, it quantifies 32 including 16 not encounter the phenomenon of the reversed majority.

5.2 Comparison with the theoretical model

Now, the objective is to make a comparison with the theoretical result. Taking into account only the previously obtained results, we note that the differences between the theoretical probabilities and the probabilities observed diverge significantly. The theoretical probabilities, in addition to their nonagreement, are higher. Indeed, the divergence between the two models, IC and IAC, do not have strange results: model IC described **the behavior less homogeneous of voters** than that generated by the assumption IAC, and always gives higher values of the voting paradoxes. Let us note that the agreement is sometimes found in other situations : the Condorcet effect is an example (see Berg and Bjurulf (1985)), but for the analysis of the propensity to manipulation, the results can be very different according to the assumption (see Lepelley, Pierron and Valognes (2000)).

Thus, it should be noted that these models must be adapted to the type of election which we consider. To allow a certain valid comparison with reality, we compare the results by taking into consideration some stylized facts which can be observed in the case of the French cantonal elections. Nevertheless, we have to maintain in mind that the situation which was modeled by Feix *et al.* (2004) is very different. All the jurisdictions have the same population, and the significant questions such as the inequalities of population between among the jurisdictions or

malapportionment of the seats among the constituencies were not treated.

Obviously, the (IC) and (IAC) models describe only extremely tied elections. It is clear for the IC in the sense that each voter chooses to vote with the same probability for one of the two political forces. But, as the assumption the IAC supposes that the votes are independent between the jurisdictions, by the law of the great numbers, the total number of the votes cast in favour of a party will present a maximum around 50% and becomes increasingly gathered in top when the number of jurisdictions increases.

Thus, we must be focused on the tightest elections to compare the empirical results, and to drop from the database the departments whose the inclination for a camp is marked.

Then, examining the results of the cantonal elections with more detail, we can take into account two issues to get a better estimate of the frequency of the paradox:

1. Some departments that always favour the same political force should be removed from the database. We consider departments that swung at least once.
2. A tightened election, when the margin between the two political coalitions is less than 10%, is a better base to compute the frequency.

Before starting the comparison, note that Feix and *al.* (2004) have, in addition, proposed a medium version of the interpretation of the definition of the referendum paradox. In fact, in their theoretical studies, they suppose that the electoral results are decided by tossing a fair coin, i.e. the probability of having certain paradoxical cases is $1/2$. Thus, they retain only half of the paradoxical situations when the two political forces obtain the same number of seats (and not the same number of voices) and/or when a party wins k seats if the total number of seats at stake is $2k + 1$. In the same way, we will follow the same logic and we will remake the calculation of the frequency by adopting this medium position.

Table 14 summarizes our findings. It provides these results for the four methods of measurement, and for the type of the elections which are taken into account. Each line displays three values: the first corresponds to the weak version of the definition of the paradox, the second corresponds to the medium version -it is the principal value of the comparison- and the last one is the frequency calculated according to the strong version.

Initially, to calculate the brut frequency according to the weak version, we keep the 93 departments to serve as a basis for analysis, however we did not retain the cases where the left or the right collects 50% of seats with a different percentage of votes, and the cases where the number of seats at stake is odd and the variation of gain in terms of seats is equal to 1. The occurrence of the paradox remains rather weak.

We exclude, thereafter, the departments which always support a political coalition in terms of seats and votes. Then there are 73 departments, among the

Table 14: The frequency of the paradox according to the four methods

	Method 1	Method 2	Method 3	Method 4
Brut frequency	<i>3.69%</i>	<i>7.37%</i>	<i>6.91%</i>	<i>7.53%</i>
	6.99%	10.60%	10.06%	12.44%
	<i>10.29%</i>	<i>13.82%</i>	<i>13.21%</i>	<i>17.36%</i>
The 73 swing departments	<i>4.70%</i>	<i>9.39%</i>	<i>8.61%</i>	<i>9.59%</i>
	8.90%	13.50%	12.72%	15.85%
	<i>13.11%</i>	<i>17.61%</i>	<i>16.83%</i>	<i>22.11%</i>
The 50 swing departments	<i>5.14%</i>	<i>10.86%</i>	<i>9.71%</i>	<i>9.14%</i>
	10.00%	16.29%	15.14%	15.00%
	<i>14.86%</i>	<i>21.71%</i>	<i>20.57%</i>	<i>20.86%</i>
Tightened elections	<i>9.96%</i>	<i>18.02%</i>	<i>17.55%</i>	<i>13.78%</i>
	18.88%	25.00%	24.90%	21.96%
	<i>27.80%</i>	<i>31.98%</i>	<i>32.24%</i>	<i>30.45%</i>
Tightened elections and the 73 swing departments	<i>10.39%</i>	<i>18.55%</i>	<i>17.92%</i>	<i>15.17%</i>
	19.48%	25.11%	25.21%	22.93%
	<i>28.57%</i>	<i>31.67%</i>	<i>32.50%</i>	<i>30.69%</i>
Tightened elections and the 50 swing departments	<i>12.14%</i>	<i>18.75%</i>	<i>17.55%</i>	<i>12.09%</i>
	24.28%	26.70%	26.33%	20.47%
	<i>36.42%</i>	<i>34.66%</i>	<i>35.11%</i>	<i>28.84%</i>

93, which swing. The result of calculation shows that the frequency of the paradox, according to the four methods, oscillates between 8,02% and 15,85% according to medium version, and between 13,11% and 22,11% according to strong version (see table 14). On the other hand, when we remove from the database only the departments which always support a political force in terms of seats, one has 350 department to study (50×7 elections). Consequently, the frequency increases, it is balanced, now, between 10% and 16,29% (according to the median version).

Secondly, we focused on the assumption according to which the variation of vote does not exceed a certain percentage of vote. Thus, we consider only the departments for which this condition of the limitation of variation of vote between the two major political forces is fulfilled. The interest is to consider only the tightened elections between the left and right parties. Then, we will combine the results obtained with those released previously, i.e. we hold account only the tightened elections which are unrolled in the departments which favour always a camp (initially the 73 departments, then he 50 departments that swing).

The occurrence of the paradox is higher and it is more closer to the theoretical result. It exceeds even the theoretical frequency by using the second and the third methods. Moreover, if one retains only the data concerning the second round (in order to eliminate the effect of the two rounds and the presence of the thirds parties) to approach the modeled situation (in order to retain only the most

disputed elections), we observed that the frequency calculated in the cantonal elections (20,47%) includes between the one given by the IAC (16,5%) and the one of the IC (20,5%). According to these data, the forecasts of the models seem completely reasonable!

These results approve the model used by Feix et al. (2004) and indicate that this can be applied to other situations of vote which are not very complicated.

6 Conclusion

Typically, the Social Choice theory attempts to show the existence of paradoxical situations, while seeking only seldom if such phenomena occur indeed in reality. Our first objective was to find a sufficiently rich electoral database, and a simple paradox which is clearly identifiable. From our point of view, and in spite of the difficulties of defining precisely the referendum paradox in an election in two rounds gathering more than two parties, this study ended with positive results. According to the paradox measurement method, the frequency oscillates between 6% and 11%, with 40% to 50% of the departments affected.

These results are to be compared with the theoretical study of Feix et al.(2004), which sought the frequency of the paradox with a priori models of vote. All in all, the theoretical probabilities and the probabilities observed are more closer if one excludes from the database the departments which always give the preference to the same camps in terms of votes and seats, and if one considers only the departments where the margin of vote between the two major coalitions does not exceed 10%.

Our results also clearly show that the paradoxical situations cannot be explained by a unique cause: the existence of triangular, the level of the turnout, the dispersion of the unequal population according to cantons, the impact of the drawing of electoral boundaries, or the existence of the two rounds instead of one. The question of finding a methodology to measure the impact as of these various elements arises. A lead could be to adapt to the case of the two rounds majority system, the methods developed by Saxon-anglo authors (Broockes (1957,1960), Grofman and al.(1997), Johnston et al. (1999), Blau (2004)) for the study of the paradox in the United States, New Zealand and in England (one round system). In particular these authors manage to distinguish three main partisan bias: turnout rate differences, impact of the size differences of population across constituencies (malapportionment) and impact of the drawing of electoral boundaries (gerrymandering), to which are added sometimes the question of the influence of a third block.

Finally, the rather high occurrence of the paradox can provoke doubt in the electoral system currently in force. Therefore, it seems to us that these results plead for change in councillor elections. The adoption of a system based on the proportional representation of the political tendencies to the level of the department makes possible to eliminate the multiple factors which disturb the election. Another solution, as for the American presidential elections, would be to attach to each canton a number of mandates proportional to its population.

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Appendix

Table 15: Departments where the paradox, in the event of equality of seats at stake, occurs according to the first approach.

Year	department	Registered voters	Seats at stake	Left		Right	
				% Vote	C	% Vote	C
2004	1 AIN	179160	22	16.85%	11	14.81%	11
	25 DOUBS	158024	18	20.24%	9	14.31%	9
	57 MOSELLE	381859	26	16.56%	13	15.33%	13
	60 OISE	246427	20	15.32%	10	15.70%	10
1998	28 EURE ET LOIR	140214	14	30.27%	7	19.34%	7
	70 HAUTE SAONE	81388	16	28.67%	8	26.33%	8
	79 DEUX SEVRES	124605	16	30.37%	8	28.14%	8
	82 TARN ET GARONNE	71590	14	26.77%	7	30.08%	7
1994	16 CHARENTE	140985	18	30.34%	9	23.75%	9
	39 JURA	96458	17	25.80%	8	21.19%	8
	70 HAUTE SAONE	88284	16	29.47%	8	27.31%	8
	71 SAONE ET LOIRE	223413	30	32.18%	15	21.87%	15
1992	4 ALPES DE HAUTE PROVENCE	53507	16	23.31%	8	31.34%	8
	24 DORDOGNE	167380	26	28.03%	13	27.30%	13
	84 VAUCLUSE	148979	12	20.60%	6	25.98%	6
	93 SEINE SAINT-DENIS	340629	20	23.21%	10	30.50%	10
1988	4 ALPES DE HAUTE PROVENCE	45107	14	27.75%	7	32.45%	7
	54 MEURTHE ET MOSELLE	245591	20	29.22%	10	29.35%	10
	58 NIEVRE	87706	16	33.44%	8	27.05%	8
	84 VAUCLUSE	156976	12	24.86%	6	34.40%	6
1985	28 EURE ET LOIR	125390	14	22.02%	7	35.46%	7
	32 GERS	62738	16	28.41%	8	27.75%	8

C : Councillors.

Table 16: Departments where the paradox, in the event of equality of seats at stake, appears according to the second approach.

Year	department	Votes cast	C	Left		Right		Others	
				% Vote	C	% Vote	C	% Vote	C
2004	1 AIN	111781	22	46.30%	11	53.70%	11	0.00%	0
	25 DOUBS	103470	18	44.29%	9	55.50%	9	0.21%	0
	57 MOSELLE	209842	26	50.38%	13	49.62%	13	0.00%	0
	60 OISE	157513	20	43.64%	10	56.36%	10	0.00%	0
1998	28 EURE ET LOIR	76967	14	44.49%	7	55.51%	7	0.00%	0
	70 HAUTE SAONE	51425	16	50.52%	8	49.48%	8	0.00%	0
	79 DEUX SEVRES	68189	16	47.37%	8	51.38%	8	1.25%	0
	82 TARN ET GARONNE	44259	14	55.20%	7	44.67%	7	0.12%	0
1994	16 CHARENTE	80415	18	50.34%	9	49.29%	9	0.38%	0

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... table 16 continued

	39	JURA	59604	17	40.52%	8	52.15%	8	0.00%	0
	70	HAUTE SAONE	58656	16	51.15%	8	48.85%	8	0.00%	0
	71	SAONE ET LOIRE	132734	30	48.12%	15	51.73%	15	0.15%	0
1992	4	ALPES DE HAUTE PROVENCE	37529	16	49.83%	8	49.38%	8	0.79%	0
	24	DORDOGNE	120663	26	49.62%	13	49.87%	13	0.50%	0
	84	VAUCLUSE	96238	12	38.49%	6	60.34%	6	1.17%	0
	93	SEINE SAINT-DENIS	178218	20	35.39%	10	64.61%	10	0.00%	0
1988	4	ALPES DE HAUTE PROVENCE	29193	14	49.55%	7	49.79%	7	0.66%	0
	54	MEURTHE ET MOSELLE	104713	20	49.29%	10	50.71%	10	0.00%	0
	58	NIEVRE	43456	16	60.42%	8	39.58%	8	0.00%	0
	84	VAUCLUSE	82549	12	49.55%	6	50.45%	6	0.00%	0
1985	28	EURE ET LOIR	84684	14	45.65%	7	54.35%	7	0.00%	0
	32	GERS	45531	16	51.58%	8	48.42%	8	0.00%	0

C : Councillors.

Table 17: departments where the paradox, in the event of equality of seats at stake, happens according to the third approach.

Year	department	Votes	C	Left		Right		EXR		
				% Vote	C	% Vote	C	% Vote	C	
2004	1	AIN	111781	22	46.30%	11	45.15%	11	8.55%	0
	25	DOUBS	103470	18	44.29%	9	46.21%	9	9.28%	0
	57	MOSELLE	209842	26	50.38%	13	44.25%	13	5.37%	0
	60	OISE	157513	20	43.64%	10	40.85%	10	15.51%	0
1998	28	EURE ET LOIR	76967	14	44.49%	7	45.91%	7	9.60%	0
	70	HAUTE SAONE	51425	16	50.52%	8	41.77%	8	7.70%	0
	79	DEUX SEVRES	68189	16	47.37%	8	48.81%	8	2.57%	0
	82	TARN ET GARONNE	44259	14	55.20%	7	39.76%	7	4.92%	0
1994	16	CHARENTE	80415	18	50.34%	9	48.17%	9	1.12%	0
	39	JURA	59604	17	40.52%	8	51.84%	8	0.31%	0
	70	HAUTE SAONE	58656	16	51.15%	8	45.00%	8	3.85%	0
	71	SAONE ET LOIRE	132734	30	48.12%	15	49.67%	15	2.06%	0
1992	4	ALPES DE HAUTE PROVENCE	37529	16	49.83%	8	46.86%	8	2.52%	0
	24	DORDOGNE	120663	26	49.62%	13	48.70%	13	1.18%	0
	84	VAUCLUSE	96238	12	38.49%	6	40.86%	6	19.48%	0
	93	SEINE SAINT-DENIS	178218	20	35.39%	10	40.96%	10	23.66%	0
1988	4	ALPES DE HAUTE PROVENCE	29193	14	49.55%	7	47.91%	7	1.88%	0
	54	MEURTHE ET MOSELLE	104713	20	49.29%	10	49.93%	10	0.77%	0
	58	NIEVRE	43456	16	60.42%	8	38.14%	8	1.44%	0
	84	VAUCLUSE	82549	12	49.55%	6	50.00%	6	0.44%	0
1985	28	EURE ET LOIR	84684	14	45.65%	7	52.42%	7	1.93%	0
	32	GERS	45531	16	51.58%	8	48.42%	8	0.00%	0

Table 18: Departments where the paradox, in the event of equality of seats at stake, occurs for seat called at the second round.

Year	department	Vote	C	Left		Right		EXR	
				% Vote	C	% Vote	C	% Vote	C
2004	23 CREUSE	10471	4	52.78%	2	47.22%	2	0.00%	0
	36 INDRE	34500	6	52.29%	3	36.46%	3	0.00%	0
	51 MARNE	84044	14	51.47%	7	42.97%	7	5.56%	0
	60 OISE	124863	16	43.02%	8	41.95%	8	15.04%	0
2001	15 CANTAL	14302	4	53.63%	2	46.37%	2	0.00%	0
	17 CHARENTE MARITIME	73102	16	46.77%	8	51.51%	8	0.00%	0
	42 LOIRE	114252	16	48.14%	8	51.86%	8	0.00%	0
	48 LOZERE	14950	6	46.05%	3	53.95%	3	0.00%	0
	56 MORBIHAN	95515	14	46.02%	7	53.98%	7	0.00%	0
	62 PAS DE CALAIS	143674	22	53.81%	11	45.23%	11	0.00%	0
	77 SEINE ET MARNE	135215	16	47.89%	8	52.11%	8	0.00%	0
	80 SOMME	70528	14	44.00%	6	40.27%	6	0.00%	0
94 VAL DE MARNE	131071	22	48.92%	11	51.08%	11	0.00%	0	
1998	16 CHARENTE	65831	16	47.73%	8	52.27%	8	0.00%	0
	19 CORREZE	41696	12	47.37%	6	52.63%	6	0.00%	0
	41 LOIR ET CHER	48985	12	41.95%	6	55.81%	6	2.24%	0
	43 HAUTE LOIRE	32429	10	43.57%	5	56.43%	5	0.00%	0
	69 RHONE	185184	22	46.70%	11	34.93%	11	18.09%	0
	70 HAUTE SAONE	33477	10	52.79%	5	43.32%	5	3.89%	0
1994	12 AVEYRON	32728	8	50.48%	4	49.52%	4	0.00%	0
	18 CHER	54706	14	44.68%	7	55.32%	7	0.00%	0
	27 EURE	75428	14	45.38%	7	49.33%	7	5.30%	0
	35 ILLE ET VILAINE	128755	20	47.21%	10	50.76%	10	0.00%	0
	37 INDRE ET LOIRE	62465	11	47.74%	5	48.87%	5	0.00%	0
	72 SARTHE	73324	14	49.76%	7	50.24%	7	0.00%	0
	80 SOMME	85748	16	48.08%	8	50.99%	8	0.93%	0
	81 TARN	59141	12	49.20%	6	50.80%	6	0.00%	0
1992	11 AUDE	53399	10	58.79%	5	41.21%	5	0.00%	0
	40 LANDES	50612	8	52.35%	4	47.65%	4	0.00%	0
	46 LOT	39415	12	48.09%	6	51.91%	6	0.00%	0
	48 LOZERE	4727	4	53.86%	2	46.14%	2	0.00%	0
	84 VAUCLUSE	96238	12	38.49%	6	40.86%	6	19.48%	0
	93 SEINE SAINT-DENIS	178218	20	35.39%	10	40.96%	10	23.66%	0
1988	1 AIN	47490	12	45.97%	6	54.03%	6	0.00%	0
	2 AISNE	54245	10	56.95%	5	43.05%	5	0.00%	0
	3 ALLIER	58755	14	52.98%	7	47.02%	7	0.00%	0
	4 ALPES DE HAUTE PROVENCE	15049	8	45.37%	4	54.63%	4	0.00%	0

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... table 18 continued

	7	ARDECHE	45375	10	47.31%	5	52.69%	5	0.00%	0
	16	CHARENTE	33255	8	53.31%	4	46.69%	4	0.00%	0
	41	LOIR ET CHER	25792	6	50.55%	3	49.45%	3	0.00%	0
	44	LOIRE ATLANTIQUE	69539	12	50.58%	6	49.42%	6	0.00%	0
	56	MORBIHAN	70421	10	47.99%	5	52.01%	5	0.00%	0
	65	HAUTES PYRENEES	33333	10	60.12%	5	39.88%	5	0.00%	0
	76	SEINE MARITIME	144576	24	55.23%	12	44.77%	12	0.00%	0
	79	DEUX SEVRES	32795	8	34.87%	4	65.13%	4	0.00%	0
	85	VENDEE	14300	2	51.09%	1	48.91%	1	0.00%	0
	86	VIENNE	49032	12	51.86%	6	48.14%	6	0.00%	0
1985	23	CREUSE	19690	6	49.14%	3	50.86%	3	0.00%	0
	33	GIRONDE	108765	14	48.39%	7	51.61%	7	0.00%	0
	45	LOIRET	56409	10	44.10%	5	55.90%	5	0.00%	0