Egalitarianism and utilitarianism in committees of representatives*

Annick Laruelle† and Federico Valenciano‡

June 11, 2008

Abstract

We address the issue of the choice of voting rule in a committee in which each member acts on behalf of a group of individuals or a constituency of different sizes. We assume that the committee of representatives makes dichotomous choices (acceptance/rejection) by vote. Given the sizes of each group, what is the most adequate voting rule for the committee? We provide answers based on each of the two principles commonly used to make normative assessments in different contexts: egalitarianism and utilitarianism. To do so, we introduce utilities in the model and adopt a normative approach.

1 Introduction

We address the following problem: Consider a committee in which each member acts on behalf of a group of individuals (or a constituency) of different sizes. Given the number of members in this committee and the size of each group represented, what is the most adequate voting rule for the committee? Our goal is to provide answers based on each of the two principles commonly used to make normative assessments in different contexts: egalitarianism and utilitarianism. The former states that an equal treatment should be given to equals, or, in utility terms, the same utility level. The latter prescribes maximizing the sum of voters' utilities. However, often the only data that are available at a design stage for the choice of voting rule are the number of groups together with

*This research is supported by the Spanish Ministerio de Ciencia y Tecnología under project SEJ2006-05455, co-funded by FEDER, and has also benefitted from Acción Integrada HF-2006-0021/EGIDE-Picasso. The second author also benefits from the Basque Government's funding to Grupos Consolidado ???.

†UFR Sciences Economiques et de Gestion, Université de Caen, rue Claude Bloch 19, F-14032 Caen Cedex, France; annick.laruelle@unicaen.fr

‡Departamento de Economía Aplicada IV, Universidad del País Vasco, Avenida Lehendakari Aguirre, 83, E-48015 Bilbao, Spain; federico.valenciano@ehu.es.

their sizes. Thus a well-founded answer for the implementation of either principle requires the model to take these data as the only input of the model. More precisely, some assumptions about the preferences within the groups represented and about their relationship with the votes of their representatives are necessary. This is done adopting a normative point of view, so to say, behind a veil of ignorance concerning individuals' actual preferences. Finally, some assumptions concerning the utilities of the people represented when they obtain or not their preferred outcome in a committee's decision are also needed. Here again, a normative approach is adopted: we do not distinguish among individuals concerning the level of utility that they can obtain. Once all these ingredients are put into the model the question of the choice of voting rule in the committee is addressed when the represented groups are sufficiently large. It turns out that egalitarianism does not restrict the choice of rule in the committee. Utilitarianism is more demanding, but can be implemented rather accurately. It is interesting to remark that all the conclusions, apart from the objective data (number of groups and their sizes) depend exclusively on two parameters concerning individuals: the marginal utility of winning a positive vote (w.r.t. to losing) and the marginal utility of winning a negative vote (w.r.t. to losing). In fact, strictly speaking it depends on a single parameter: the ratio of these marginal utilities.

Some related work deserves to be mentioned. Barberà and Jackson (2006) consider a very general setup where each representative's vote in the committee is a function of the preferences within his/her group and a probability distribution over these preference profiles is known. They address the question of the decision in the committee that maximizes the expected aggregated utility. Fleurbaey (2007) also addresses the choice of voting rule from a welfarist approach, where utilities are rather interpreted as "measuring the social value of i's situation for the observer". Beisbart and Bovens (2007) adopt a similar approach but claim that the rule cannot depend on the issue which is at stake[1]. Our approach is also resolutely normative-oriented at a design stage: individuals' actual utilities are ignored as unknowable. Our model directly adopts in this respect a 'behind of a veil of ignorance' point of view according to which an individual may only be of two 'types' on each issue: either a supporter or a rejecter. This same point of view leads us to assume all (so restricted) preference profiles equally probable instead of dealing with arbitrary probability distributions over arbitrary utility profiles or with profiles of probability distributions over personal utilities. The closest work is Chapter 3 in Laruelle and Valenciano (2008), where we address these issues in less general conditions. When the conditions on preferences and decision-making are further conveniently specified some of the conclusions of the works mentioned are very close to some of those obtained here.

The rest of the paper is organized as follows. We first introduce the basic notation concerning voting rules (Section 2) and introduce the assumptions concerning the utilities of the individuals in a vote (Section 3). In Section 4 *a priori* expected utilities

[1] In Barberà and Jackson (2006) or Fleurbaey (2007), the optimal voting rule depends on the utilities that voters obtain in the issue.

in a vote are calculated, while in Section 5 *a priori* expected utilities in a committee of representatives. The choice of voting rule in a committee of representatives is addressed from the egalitarianism (Section 6) and from the utilitarianism (Section 7) point of views. Finally, Section 8 contains some concluding remarks.

2 Voting rules

Let $N = \{1, 2, .., n\}$ be the set of *seats*' labels. Voters are labelled by their seats' labels. A *vote configuration* is a possible result of a vote, that lists the vote cast by the voter occupying each seat. Under the assumption that voters may only vote 'yes' or 'no', there are 2^n possible vote configurations. We represent any *vote configuration* by the set of the yes-voters $S \subseteq N$ (the voters in $N \backslash S$ voting 'no'). The cardinal of S is denoted by s or $\#S$. For $i \in S$, we write $S \backslash i$ instead of $S \backslash \{i\}$.

An N-*voting rule* is specified by the set $\mathcal{W} \subseteq 2^N$ of vote configurations that would lead to the passage of a proposal. This set of *winning configurations* is assumed to satisfy the following requirements: (i) $N \in \mathcal{W}$; (ii) $\emptyset \notin \mathcal{W}$; (iii) If $S \in \mathcal{W}$, then $T \in \mathcal{W}$ for any T containing S; and (iv) If $S \in \mathcal{W}$ then $N \backslash S \notin \mathcal{W}$. The first two conditions are Pareto efficiency conditions, while the third is a condition of monotonicity. The last condition prevents the passage of a proposal and its negation if they were supported by S and $N \backslash S$, respectively. When only the first three conditions are satisfied the rule is said to be *improper*.

A voting rule is *anonymous* (or *symmetric*) if a voting configuration being winning or not only depends on its size. They can be represented by q-*majority rules*, denoted by $\mathcal{W}^{qM}$, where a proposal is passed if the proportion of votes in favor of the proposal is greater than q. That is,

$$\mathcal{W}^{qM} = \{S \subseteq N : \frac{s}{n} > q\},$$

with $\frac{1}{2} \leq q < 1$. Special cases are the *simple majority* rule ($q = \frac{1}{2}$) denoted by $\mathcal{W}^{SM}$, and the *unanimity* rule ($q = 1 - \varepsilon$, $0 < \varepsilon < \frac{1}{n}$).

A *weighted majority rule,* denoted by $\mathcal{W}^{(w,Q)}$, is specified by a system of positive *weights* $w = (w_1, .., w_n)$, and a *quota* $Q > 0$, so that the final result is 'yes' if the sum of the weights in favor of the proposal is larger than the quota, that is:

$$\mathcal{W}^{(w,Q)} = \{S \subseteq N : \sum_{i \in S} w_i > Q\},$$

which can alternatively be specified in terms of the *relative quota* $q := \frac{Q}{\sum_{j \in N} w_j}$, as

$$\mathcal{W}^{(w,q)} = \{S \subseteq N : \frac{\sum_{i \in S} w_i}{\sum_{j \in N} w_j} > q\}.$$

3 Utility of a vote

We deal here with what we have called elsewhere a 'take-it-or-leave-it committee'[2] to which proposals are submitted, and the committee can only accept or reject them by a vote. In particular no room is left for negotiating or modifying the proposals. We assume that no voter is indifferent between acceptance and rejection on any issue, and voting is not costly. In these conditions, we assume that voters are rational in the sense that they vote 'yes' or 'not' on an issue only depending on whether their utility is higher if the proposal is accepted or higher if it is rejected. Thus, if S denotes the resulting vote configuration on a given issue, voter i's vote is 'yes' (i.e. $i \in S$) if and only if voter i's utility in case of acceptance, denoted by $u_{i^+}(Acc) = A^{i+}$, is greater than in case of rejection, denoted by $u_{i^+}(Rej) = R^{i+}$. As the proposal is accepted if $S \in \mathcal{W}$, and rejected if $S \notin \mathcal{W}$, we use the following notation, the superscript '+' indicating that i is a 'yes'-voter (i.e. $i \in S$):

$$u_{i^+}(\mathcal{W}, S) = \begin{cases} u_{i^+}(Acc) = A^{i+} \text{ if } S \in \mathcal{W}, \\ u_{i^+}(Rej) = R^{i+} \text{ if } S \notin \mathcal{W}, \end{cases}$$

with $A^{i+} > R^{i+}$. Similarly if a voter j is against the proposal ($j \notin S$), her or his utility depends on whether the proposal is accepted or rejected. In this case we write

$$u_{j^-}(\mathcal{W}, S) = \begin{cases} u_{j^-}(Acc) = A^{j-} \text{ if } S \in \mathcal{W}, \\ u_{j^-}(Rej) = R^{j-} \text{ if } S \notin \mathcal{W}, \end{cases}$$

with $A^{j-} < R^{j-}$. We assume two forms of 'anonymity'. First, concerning the issues, we assume that the four possible utilities for a voter (i.e. $A^{i+}, R^{i+}, A^{j-}, R^{j-}$) are the same for every issue. Second, 'anonymity' concerning the voters: we assume that the intensity of preferences is identical for all voters. This sort of 'anonymity of issues' as well as the 'anonymity of voters' can be justified from a normative point of view at a design stage with a 'veil of ignorance' argument, as it is not known in advance the importance that each voter will give to each issue. We can then drop 'i' and 'j' in the superindices and write just A^+ instead of A^{i+}, R^+ instead of R^{i+}, etc.

We can summarize the above assumptions like this: If a decision is made by means of voting rule $\mathcal{W}$ and the resulting vote configuration is S the utility of voter i is given by

$$u_i(\mathcal{W}, S) = \begin{cases} A^+ \text{ if } i \in S \in \mathcal{W}, \\ R^+ \text{ if } i \in S \notin \mathcal{W}, \\ R^- \text{ if } i \notin S \notin \mathcal{W}, \\ A^- \text{ if } i \notin S \in \mathcal{W}, \end{cases} \tag{1}$$

with $A^+ > R^+$ and $A^- < R^-$.

[2] See Laruelle and Valenciano (2008) for a distinction between this type of committee and a 'bargaining committee' with capacity to modify proposals and bargain in search of agreement.

Observe that from the point of view of any voter, comparing A^+ or R^+ (her utilities in case of acceptance or rejection of *a proposal she supports*) with A^- or R^- (her utilities in case of acceptance or rejection of *a proposal she rejects*) requires *comparing the utility for different issues*, an issue the voter favors and one the voter is against. Although different assumptions can be made in this respect[3], we make no further assumptions. As we will see, in the model we presently consider the relevant data are the differences

$$\Delta^+ \quad : \quad = A^+ - R^+ \quad \text{and} \quad \Delta^- := R^- - A^- \tag{2}$$

$$\text{with } \Delta^+ \quad > \quad 0, \text{ and } \Delta^- > 0. \tag{3}$$

that is to say, the marginal utility of winning a vote (w.r.t. losing) for an affirmative voter (Δ^+), and the marginal utility of winning a vote (w.r.t. losing) for a negative voter (Δ^-).

As we introduce presently risk in the model, we assume voters' preferences to be Von Neumann-Morgenstern's (1944) or expected utility preferences. Such preferences are defined up to a positive affine transformation and we need to make comparisons between voters' expectations. Thus, any recommendation to be made on the basis of $u_i(\mathcal{W}, S)$ should not be altered if the utilities are replaced by

$$u_i'(\mathcal{W}, S) = au_i(\mathcal{W}, S) + b,$$

for some $a > 0$ and some b, *as long as this is done with the same a and b for any voter i*. Observe no conflict arises in this case for *exact* egalitarianism, because it is equivalent to require for all i, j:

$$u_i'(\mathcal{W}, S) = u_j'(\mathcal{W}, S) \quad \text{or} \quad u_i(\mathcal{W}, S) = u_j(\mathcal{W}, S).$$

Also for *exact* utilitarianism, as it is equivalent to maximize

$$\sum_{i \in N} u_i'(\mathcal{W}, S) \quad \text{or} \quad \sum_{i \in N} u_i(\mathcal{W}, S).$$

The problem may appear with comparisons between voters' utilities in case of inequality. Absolute comparisons, that is, the differences

$$u_i'(\mathcal{W}, S) - u_j'(\mathcal{W}, S) = a(u_i(\mathcal{W}, S) - u_j(\mathcal{W}, S)) \neq u_i(\mathcal{W}, S) - u_j(\mathcal{W}, S),$$

are not altered by b, but by a (unless $a = 1$), while for relative comparisons (i.e. quotients), in general

$$\frac{u_i'(\mathcal{W}, S)}{u_j'(\mathcal{W}, S)} = \frac{au_i(\mathcal{W}, S) + b}{au_j(\mathcal{W}, S) + b} \neq \frac{u_i(\mathcal{W}, S)}{u_j(\mathcal{W}, S)},$$

[3]For instance, it may seems reasonable to have $A^+ \geq A^-$ or $R^- \geq R^+$. This assumption is not necessary here. In Laruelle and Valenciano (2008) we assume (once translated into the current setting and notation) that $R^+ = A^-$. Alternatively, one can assume that when proposals are rejected the *status quo* prevails, as in Barberà and Jackson (2007) and Beisbart and Bovens (2007), who consider different utilities for each voter and assume (in our notation) $R_i^- = R_i^+$, while Fleurbaey (2007) assumes instead $A_i^+ = R_i^-$ and $R_i^+ = A_i^-$.

so that ratios are altered by a and b (unless $b = 0$). In order to make comparisons independent of a and b, we will divide absolute differences by the difference between the maximal (u_{Max}) and minimal (u_{Min}) utilities within the range of feasible utilities. In this way we have, for whatever a and b, :

$$\frac{u'_i(\mathcal{W}, S) - u'_j(\mathcal{W}, S)}{u'_{Max} - u'_{Min}} = \frac{u_i(\mathcal{W}, S) - u_j(\mathcal{W}, S)}{u_{Max} - u_{Min}}. \quad (4)$$

For relative comparisons we take quotients of differences with the minimal utility, so that for all a and b, we have

$$\frac{u'_i(\mathcal{W}, S) - u'_{Min}}{u'_j(\mathcal{W}, S) - u'_{Min}} = \frac{u_i(\mathcal{W}, S) - u_{Min}}{u_j(\mathcal{W}, S) - u_{Min}}. \quad (5)$$

4 Expected utility of a vote

Each voter is of two possible 'types': a 'yes'-voter (i.e. one whose utilities in case of acceptance and rejection are given respectively by A^+ and R^+), and a 'no'-voter (i.e. one whose utilities in case of acceptance and rejection are given respectively by A^- and R^-). Each voter learns her/his type once the issue is known. Consistently with the normative approach, we assume that each voter is either type with probability 1/2

$$P(i \in S) = P(i \notin S) = \frac{1}{2} \quad \text{for all } i \in N,$$

independently of the others' types. The permits to derive a 'common prior' on any possible vote configuration. If $p(S)$ denotes the probability of occurrence of vote configuration S, we have :

$$p(S) = \frac{1}{2^n} \quad \text{for all } S \subseteq N. \quad (6)$$

The probability distribution p is such that all preference/vote configurations are equally probable. As all the calculations that follow are made assuming this normative prior, we should use terms as '*a priori* expected utility', or '*a priori* egalitarianism/utilitarianism', etc., nevertheless we often omit the term to avoid a heavy repetition.

The following Lemma gives the expected utility of a voter under the above assumptions.

Lemma 1 *Assuming (1) and (6), the expected utility of voter i is given by*

$$E_p\left[u_i(\mathcal{W}, S)\right] = \frac{1}{2}(R^+ + R^-) + \frac{1}{2}(\Delta^+ - \Delta^-) \sum_{S:S\in\mathcal{W}} \frac{1}{2^n} + \frac{1}{2}(\Delta^+ + \Delta^-) \sum_{\substack{S:i\in S\in\mathcal{W} \\ S\setminus i\notin\mathcal{W}}} \frac{1}{2^n}. \quad (7)$$

Proof. By (1), the expected utility of voter i is given by

$$\begin{aligned} E_p\left[u_i(\mathcal{W},S)\right] &= A^+P(i\in S\in\mathcal{W})+R^+P(i\in S\notin\mathcal{W}) \\ +A^-P(i &\notin S\in\mathcal{W})+R^-P(i\notin S\notin\mathcal{W}), \end{aligned}$$

where $P(i\in S\in\mathcal{W})$ is the probability that i votes 'yes' and the proposal is accepted (i.e. '$i\in S$ & $S\in\mathcal{W}$'), etc. In view of the three following equalities:

$$\begin{aligned} P(i &\in S\notin\mathcal{W}) = P(i\in S)-P(i\in S\in\mathcal{W}) \\ P(i &\notin S\in\mathcal{W}) = P(S\in\mathcal{W})-P(i\in S\in\mathcal{W}), \\ P(i &\notin S\notin\mathcal{W}) = P(i\notin S)-P(i\notin S\in\mathcal{W}) \end{aligned}$$

the above expression can be rewritten as

$$E_p\left[u_i(\mathcal{W},S)\right]=(A^+-R^++R^--A^-)P(i\in S\in\mathcal{W})+R^+P(i\in S)$$

$$-(R^--A^-)P(S\in\mathcal{W})+R^-P(i\notin S).$$

By (6), using notation (2), we have:

$$E_p\left[u_i(\mathcal{W},S)\right]=(\Delta^++\Delta^-)\sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n}-\Delta^-\sum_{S:S\in\mathcal{W}}\frac{1}{2^n}+\frac{1}{2}(R^++R^-). \tag{8}$$

Note that

$$\begin{aligned} \sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n} &= \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}}\frac{1}{2^n}+\sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\in\mathcal{W}}}\frac{1}{2^n}=\sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}}\frac{1}{2^n}+\sum_{T:i\notin T\in\mathcal{W}}\frac{1}{2^n} \\ &= \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}}\frac{1}{2^n}+\sum_{S:S\in\mathcal{W}}\frac{1}{2^n}-\sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n}. \end{aligned}$$

Therefore

$$\sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n}=\frac{1}{2}\sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}}\frac{1}{2^n}+\frac{1}{2}\sum_{S:S\in\mathcal{W}}\frac{1}{2^n}.$$

Then substituting this in (8) we have (7). ■

This Lemma tells that the expected utility of each voter is the sum of three terms, one which is independent of the rule, one which depends on the rule but is common to all voters, and one which may differ from voter to voter. In the terms that depends on the rule appears Coleman's (1971) *a priori* probability of a proposal being accepted, that we denote $\alpha(\mathcal{W})$:

$$\alpha(\mathcal{W})=\sum_{S:S\in\mathcal{W}}\frac{1}{2^n}, \tag{9}$$

while in the term that is specific to voter i to voter appears voter i's Banzhaf (1965) index for rule $\mathcal{W}$, that we denoted $Bz_i(\mathcal{W})$:

$$Bz_i(\mathcal{W}) = \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}} \frac{1}{2^{n-1}}. \tag{10}$$

Voter i's expected utility given by (7) can thus be rewritten as:

$$E_p\left[u_i(\mathcal{W},S)\right] = \frac{1}{2}(R^+ + R^-) + \frac{1}{2}(\Delta^+ - \Delta^-)\alpha(\mathcal{W}) + \frac{1}{4}(\Delta^+ + \Delta^-)Bz_i(\mathcal{W}). \tag{11}$$

We recall here some inequalities for these indices that will be used later:

$$Bz_i(\mathcal{W}) = \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}} \frac{1}{2^{n-1}} \leq \sum_{S:S\in\mathcal{W}} \frac{1}{2^{n-1}} = 2\sum_{S:S\in\mathcal{W}} \frac{1}{2^n} = 2\alpha(\mathcal{W}). \tag{12}$$

and

$$\alpha(\mathcal{W}) = \sum_{S:S\in\mathcal{W}} \frac{1}{2^n} \leq \frac{1}{2}, \tag{13}$$

which would not necessary hold for improper rules (recall that for improper rules, $\#\mathcal{W}$ may be larger than 2^{n-1}).

5 Utilities in a committee of representatives

Now consider the case of a committee of representatives in which each member acts on behalf of a group of different size. Let $N = \{1, 2, ..., n\}$ label the members of the committee that makes decisions by means of a voting rule $\mathcal{W}_N$. Each $i \in N$ represents the m_i individuals of group M_i. Groups are supposed to be pairwise disjoint. Let $M := \cup_{i\in N} M_i$ be the set of all individuals represented, and m its number. If each representative i is assumed to follow the majority opinion in group M_i on every issue, the situation may be modelled by an M-voting rule among individuals in M that we denote $\mathcal{W}_M$ in the following way. For any vote configuration among the individuals $S \subseteq M$, and each $i \in N$, let S_i be the intersection

$$S_i = S \cap M_i,$$

which determines representative i's vote in the committee of representatives. That is, $i \in N$ votes 'yes' if

$$\#S_i > \frac{m_i}{2}.$$

Thus, if $S \subseteq M$ is the vote configuration in M, the resulting vote configuration in the committee of representatives is

$$S_N = \{i \in N : \#S_i > \frac{m_i}{2}\},$$

and vote configuration $S \subseteq M$ is winning if $S_N \in \mathcal{W}_N$. Thus the decision-making procedure by M can be described by means of M-voting rule

$$\mathcal{W}_M = \{S \subseteq M : S_N \in \mathcal{W}_N\}.$$

What follows is a discussion about the choice of the voting rule $\mathcal{W}_N$ in the committee from the points of view of egalitarianism and utilitarianism, but focused on the utilities of the individuals represented (i.e. in M). Two interpretations are possible. First, the conclusions apply to a situation in which actual decision-making follows these two steps, so that each representative is like a mechanical transmitter of the choice of the majority in her group. In this case, rule $\mathcal{W}_M$ is an exact description of the actual decision-making, and the usual notation is that of a 'composite voting rule':

$$\mathcal{W}_M = \mathcal{W}_N[\mathcal{W}_{M_1}^{SM}, ..., \mathcal{W}_{M_n}^{SM}].$$

Second, such a two-step decision-making procedure can be taken also as a model of a situation in which only the committee of representatives actually exists, and their following the majority in their respective group is an approximation that allows for examining the effects (*a priori*) of the chosen voting rule from the point of view of the people represented.

We assume that individuals *in* M have expected utility preferences with utilities given by (1) with $\mathcal{W} = \mathcal{W}_M$. We also assume that individals have the same probability of being of both types, so that all preference/vote configurations *in* M are equally probable, and denote by p the probability distribution such that $p(S) = \frac{1}{2^m}$, for all $S \subseteq M$. In the following, two approximations concerning the composite rule $\mathcal{W}_M = \mathcal{W}_N[\mathcal{W}_{M_1}^{SM}, ..., \mathcal{W}_{M_n}^{SM}]$ will be used (see e.g. Laruelle and Valenciano (2008)). First, if all m_i are large enough, then for all $i \in N$ and $k \in M_i$:

$$Bz_k(\mathcal{W}_M) \simeq \sqrt{\frac{2}{\pi m_i}} Bz_i(\mathcal{W}_N). \tag{14}$$

It is also easy to see that (in fact equality holds if all m_i are odd)

$$\alpha(\mathcal{W}_M) \simeq \alpha(\mathcal{W}_N). \tag{15}$$

6 Egalitarianism

Egalitarianism pleads for an equal treatment of equals behind a veil of ignorance. In terms of the above two-stage model, the egalitarian principle is satisfied a priori if the expected utility of any two individuals in M is the same, irrespective of what group they belong to. That is, if

$$E_p[u_k(\mathcal{W}_M, S)] = E_p[u_l(\mathcal{W}_M, S)], \quad \text{for all } k, l \in M.$$

Given (11), the egalitarianism is satisfied if and only if

$$Bz_k(\mathcal{W}_M) = Bz_l(\mathcal{W}_M). \tag{16}$$

Note that this is the requirement usually made in the voting power literature, but in that case it relies on the argument that (i) what matters is 'voting power', and (ii) 'voting power' = Banzhaf index. Here this is just a necessary and sufficient condition for egalitarianism in terms of utilities.

Using (14), the equality (16) is achieved if

$$\frac{Bz_i(\mathcal{W}_N)}{\sqrt{m_i}} = \frac{Bz_j(\mathcal{W}_N)}{\sqrt{m_j}} \text{ for any } i, j \in N,$$

a result which is known in the literature as the (first) square root rule. Various works deal with looking for the rule that satisfies this condition (see Laruelle and Widgrén (1998), Leech (2002), Slomczynski and Zyczkowski (2006)).

A natural question that arises then is how far is one from egalitarianism? Here appears the difference with the power index approach, comparisons are made between expected utilities ($E_p\left[u_k(\mathcal{W}_M, S)\right]$), and *not* between Banzhaf indices interpreted as absolute measures of a priori voting power.

When all the groups are sufficiently large, the following proposition gives some significant bounds for (4) and (5). In our model, we have

$$u_{Max} = Max\{A^+, R^+, A^-, R^-\} = Max\{A^+, R^-\}$$

and

$$u_{Min} = Min\{A^+, R^+, A^-, R^-\} = Min\{A^-, R^+\}.$$

Proposition 2 *Let $\mathcal{W}_M$ be the composite rule $\mathcal{W}_M = \mathcal{W}_N[\mathcal{W}_{M_1}^{SM}, ..., \mathcal{W}_{M_n}^{SM}]$, then, if all m_i are large enough to consider (14) and (15) as good approximations, we have*

$$\frac{\left|E_p\left[u_k(\mathcal{W}_M, S)\right] - E_p\left[u_l(\mathcal{W}_M, S)\right]\right|}{u_{Max} - u_{Min}} \leq \frac{1}{2}\xi \tag{17}$$

and

$$\frac{E_p\left[u_k(\mathcal{W}_M, S)\right] - u_{Min}}{E_p\left[u_l(\mathcal{W}_M, S)\right] - u_{Min}} \leq 1 + \xi, \tag{18}$$

where ξ is given by

$$\xi := \sqrt{\frac{2}{\pi Min_{i \in N} m_i}}. \tag{19}$$

Proof. In view of Lemma 1 we have (assuming $Bz_k(\mathcal{W}_M) > Bz_l(\mathcal{W}_M)$):

$$\begin{aligned}
& \frac{|E_p[u_k(\mathcal{W}_M,S)] - E_p[u_l(\mathcal{W}_M,S)]|}{u_{Max} - u_{Min}} \\
= & \frac{1}{4}\frac{\Delta^+ + \Delta^-}{Max\{A^+,R^-\} - Min\{R^+,A^-\}}(Bz_k(\mathcal{W}_M) - Bz_l(\mathcal{W}_M)) \\
\leq & \frac{1}{4}\frac{\Delta^+ + \Delta^-}{Max\{\Delta^+,\Delta^-\}}(Bz_k(\mathcal{W}_M) - Bz_l(\mathcal{W}_M)) \\
\leq & \frac{1}{2}Bz_k(\mathcal{W}_M).
\end{aligned}$$

Now if $k \in m_j$, assuming all m_i large enough to consider that (14) is a good approximation, the last term can be rewritten

$$\frac{1}{2}Bz_k(\mathcal{W}_M) \simeq \frac{1}{2}\sqrt{\frac{2}{\pi m_j}}Bz_j(\mathcal{W}_N) \leq \frac{1}{2}\xi,$$

which leads to (17).

Now, again assuming $Bz_k(\mathcal{W}_M) > Bz_l(\mathcal{W}_M)$, we have

$$\begin{aligned}
& \frac{E_p[u_k(\mathcal{W}_M,S)] - u_{Min}}{E_p[u_l(\mathcal{W}_M,S)] - u_{Min}} \\
= & \frac{\bar{u}(\mathcal{W}_M) + \frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M) - Min\{R^+,A^-\}}{\bar{u}(\mathcal{W}_M) + \frac{1}{4}(\Delta^+ + \Delta^-)Bz_l(\mathcal{W}_M) - Min\{R^+,A^-\}} \\
\leq & \frac{\bar{u}(\mathcal{W}_M) + \frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M) - Min\{R^+,A^-\}}{\bar{u}(\mathcal{W}_M) - Min\{R^+,A^-\}} \\
= & 1 + \frac{\frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M)}{\bar{u}(\mathcal{W}_M) - Min\{R^+,A^-\}} \\
= & 1 + \frac{\frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M)}{\frac{1}{2}(\Delta^+ - \Delta^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+,A^-\}} \\
= & 1 + \frac{\frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M)}{\frac{1}{2}(\Delta^+ + \Delta^-)\alpha(\mathcal{W}_M) + (A^- - R^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+,A^-\}}.
\end{aligned}$$

In order to find an upper bound of this expression, let us show that

$$(A^- - R^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+,A^-\} \geq 0.$$

If $Min\{R^+, A^-\} = A^-$, then

$$\begin{aligned}
& (A^- - R^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+, A^-\} \\
= \; & R^-(\frac{1}{2} - \alpha(\mathcal{W}_M)) + \frac{1}{2}R^+ - \frac{1}{2}A^- - A^-(\frac{1}{2} - \alpha(\mathcal{W}_M)) \\
= \; & (R^- - A^-)(\frac{1}{2} - \alpha(\mathcal{W}_M)) + \frac{1}{2}(R^+ - A^-) \geq 0 \text{ as } R^- - A^- = \Delta^- > 0.
\end{aligned}$$

If $Min\{R^+, A^-\} = R^+$, then

$$\begin{aligned}
& (A^- - R^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+, A^-\} \\
= \; & R^-(\frac{1}{2} - \alpha(\mathcal{W}_M)) - \frac{1}{2}R^+ + A^-\alpha(\mathcal{W}_M) \\
= \; & R^-(\frac{1}{2} - \alpha(\mathcal{W}_M)) - R^+(\frac{1}{2} - \alpha(\mathcal{W}_M)) + \alpha(\mathcal{W}_M)(A^- - R^+) \\
= \; & (R^- - R^+)(\frac{1}{2} - \alpha(\mathcal{W}_M)) + \alpha(\mathcal{W}_M)(A^- - R^+) \geq 0 \text{ as } R^- > A^- \geq R^+.
\end{aligned}$$

Thus, in both cases we have

$$\frac{E_p\left[u_k(\mathcal{W}_M, S)\right] - u_{Min}}{E_p\left[u_l(\mathcal{W}_M, S)\right] - u_{Min}} \leq 1 + \frac{\frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M)}{\frac{1}{2}(\Delta^+ + \Delta^-)\alpha(\mathcal{W}_M)} = 1 + \frac{1}{2}\frac{Bz_k(\mathcal{W}_M)}{\alpha(\mathcal{W}_M)}.$$

Now if $k \in m_j$, assuming all m_i large enough to consider (14) and (15) good approximations, using (12) and we have:

$$1 + \frac{1}{2}\frac{Bz_k(\mathcal{W}_M)}{\alpha(\mathcal{W}_M)} \simeq 1 + \frac{1}{2}\frac{Bz_k(\mathcal{W}_N)}{\alpha(\mathcal{W}_N)}\sqrt{\frac{2}{\pi m_i}} \leq 1 + \xi.$$

■

As a result we have a somewhat surprising conclusion:

Claim 3 *In the a priori model of a committee of representatives, any voting rule $\mathcal{W}_N$ basically satisfies egalitarianism at the level of the people represented as long as all groups are sufficiently large.*

Thus, in the light of the present model, the egalitarian principle seems of no consequence after all for the choice of the voting rule in a take-it-or-leave-it committee of representatives. The reason is that according to the underlying model in which each individual independently has an equal probability of being a supporter or a rejecter, even the expectations of winning a vote of an individual of a group whose representative was a null voter in the committee are rather close to that of one of a group whose representative was a dictator.

This conclusion may be substantively different from the claim of the voting power approach, even if we choose $A^+ = R^- = 1$, and $R^+ = A^- = -1$ so that by (2) and (11) we obtain

$$E_p\left[u_k(\mathcal{W}_M, S)\right] = Bz_k(\mathcal{W}_M).$$

This is illustrated on an example taken from Laruelle and Valenciano (2008) for the 2004 European Council of Ministers:

Example: The Luxembourgian citizens (lu) have the highest Banzhaf index, while Latvian citizens (la) have the lowest, with the following figures:

$$\begin{aligned} E_p\left[u_{la}(\mathcal{W}_M, S)\right] &= Bz_{la}(\mathcal{W}_M) = 0.00000446, \\ E_p\left[u_{lu}(\mathcal{W}_M, S)\right] &= Bz_{lu}(\mathcal{W}_M) = 0.0000101. \end{aligned}$$

We obtain the following ratios:

$$\frac{Bz_{lu}(\mathcal{W}_M)}{Bz_{la}(\mathcal{W}_M)} = \frac{0.0000101}{0.0000044} = 2.27,$$

while ($u_{Max} = 1$ and $u_{Min} = -1$)

$$\frac{E_p\left[u_{lu}(\mathcal{W}_M, S)\right] - u_{Min}}{E_p\left[u_{la}(\mathcal{W}_M, S)\right] - u_{Min}} = \frac{0.0000101 + 1}{0.0000044 + 1} = 1.000006.$$

Thus, according to traditional voting power approach, where the Banzhaf index is considered as an absolute measure of a priori voting power, a Luxemburgian citizen's 'voting power' is more than double that of a Latvian citizen's. This ratio of 2.27 : 1 becomes very close to 1 : 1 when we compare expected utilities. Note that for absolute comparisons the maximal difference between expected utilities is also negligible with respect to the rang of utilities (of the order of 0.0028%).

7 Utilitarianism

In the current model implementing utilitarianism means choosing a voting rule $\mathcal{W}_N$ that maximizes the aggregated expected utility in M, i.e. solving the problem:

$$Max \sum_{k \in M} E_p\left[u_k(\mathcal{W}_M, S)\right].$$

Or equivalently, as aggregation and expectation permute, a voting rule that maximizes the expected aggregated utility, i.e. solves the problem:

$$Max\ E_p[\sum_{k \in M} u_k(\mathcal{W}_M, S)].$$

Therefore the point is to choose $\mathcal{W}_N$ so as to maximize the latter expectation. Note that while each M-vote configuration determines an N-vote configuration, a same N-vote configuration may result from different M-vote configurations. Thus the utilitarian-best rule consists of making, for each vote configuration in the committee, the decision for which this expectation is the highest. More precisely, if the vote configuration in the committee is $C \subseteq N$, the best decision is to accept the proposal if

$$E_p[\sum_{k\in M} u_k \mid C \text{ \& } Acc] > E_p[\sum_{k\in M} u_k \mid C \text{ \& } Rej]. \tag{20}$$

In order to calculate these expectations we need the aggregated expected utility in each group in either case (accepting or rejecting) for each vote configuration in the committee. If the vote configuration in the committee is C, then $i \in C$ (i.e. M_i's representative votes 'yes') when a majority in group M_i supports a 'yes', while if $i \in N \setminus C$ (i.e. M_i's representative supports a 'no') when no majority in group M_i supports a 'yes'. Then from the possibility of permuting aggregation and expectation the following stems straightforwardly.

Lemma 4 *For any $i \in N$, the aggregated expected utility in group M_i, given that the majority in M_i votes 'yes' and the proposal is accepted (resp., rejected), is given, respectively, by*

$$E_p[\sum_{k\in M_i} u_k \mid \#S_i > \frac{m_i}{2} \text{ \& } Acc]$$
$$= A^+ E_p[\#S_i \mid \#S_i > \frac{m_i}{2}] + A^- E_p[\#(M_i \setminus S_i) \mid \#S_i > \frac{m_i}{2}],$$
$$E_p[\sum_{k\in M_i} u_k \mid \#S_i > \frac{m_i}{2} \text{ \& } Rej]$$
$$= R^+ E_p[\#S_i \mid \#S_i > \frac{m_i}{2}] + R^- E_p[\#(M_i \setminus S_i) \mid \#S_i > \frac{m_i}{2}];$$

while the aggregated expected utility in group M_i, given that the majority in M_i does not vote 'yes' and the proposal is accepted (resp., rejected), is given, respectively, by

$$E_p[\sum_{k\in M_i} u_k \mid \#S_i \leq \frac{m_i}{2} \text{ \& } Acc]$$
$$= A^+ E_p[\#S_i \mid \#S_i \leq \frac{m_i}{2}] + A^- E_p[\#(M_i \setminus S_i) \mid \#S_i \leq \frac{m_i}{2}],$$
$$E_p[\sum_{k\in M_i} u_k \mid \#S_i \leq \frac{m_i}{2} \text{ \& } Rej]$$
$$= R^+ E_p[\#S_i \mid \#S_i \leq \frac{m_i}{2}] + R^- E_p[\#(M_i \setminus S_i) \mid \#S_i \leq \frac{m_i}{2}].$$

The next lemma[4] gives approximations of the expected numbers of voters voting 'yes' and of voters voting 'no' that appear in the preceding expressions.

[4]For a proof of these fromulae, based on Stirling approximation, see e.g. Laruelle and Valenciano (2008).

Lemma 5 *For any $i \in N$, if m_i is large enough, the expected numbers of 'yes'-voters and of 'no'-voters in group M_i, given that the majority in M_i votes 'yes' (respectively 'no'), can be approximated by*

$$E_p[\#S_i \mid \#S_i > \frac{m_i}{2}] = E_p[\#(M_i \setminus S_i) \mid \#S_i \leq \frac{m_i}{2}] \simeq \frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}},$$

$$E_p[\#(M_i \setminus S_i) \mid \#S_i > \frac{m_i}{2}] = E_p[\#S_i \mid \#S_i \leq \frac{m_i}{2}] \simeq \frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}}.$$

Thus when a majority in group M_i votes 'yes' (and consequently M_i's representative votes 'yes') the aggregated expected utility in this group *if the decision in the committee is accepting the proposal* is (with good approximation for m_i large enough):

$$E_p[\sum_{k \in M_i} u_k \mid \#S_i > \frac{m_i}{2} \ \& \ Acc] \simeq A^+ \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}}\right) + A^- \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}}\right);$$

while *if the decision in the committee is rejecting the proposal,* the aggregated expected utility in group M_i is,

$$E_p[\sum_{k \in M_i} u_k \mid \#S_i > \frac{m_i}{2} \ \& \ Rej] \simeq R^+ \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}}\right) + R^- \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}}\right).$$

Similar approximations can be done for the case in which no majority in M_i supports the proposal.

Now, aggregating across all groups we have that for a given vote configuration in the committee $C \subseteq N$, the aggregated expected utility in M *if the committee accepts the proposal*, is (with great approximation for the m_i's large enough)

$$\begin{aligned} E_p[\sum_{k \in M} u_k \mid C \ \& \ Acc] \simeq & \sum_{i \in C} \left(A^+ \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}}\right) + A^- \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}}\right)\right) \\ & + \sum_{i \in N \setminus C} \left(A^+ \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}}\right) + A^- \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}}\right)\right) \\ & = (A^+ + A^-)\frac{m}{2} + \frac{A^+ - A^-}{\sqrt{2\pi}} \left(\sum_{i \in C} \sqrt{m_i} - \sum_{i \in N \setminus C} \sqrt{m_i}\right); \end{aligned} \tag{21}$$

while *if the proposal is rejected* the aggregated expected utility is

$$E_p[\sum_{k \in M} u_k \mid C \ \& \ Rej] \simeq \sum_{i \in C} \left(R^+ \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}}\right) + R^- \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}}\right)\right)$$

$$+\sum_{i\in N\setminus C}\left(R^{+}\left(\frac{m_i}{2}-\sqrt{\frac{m_i}{2\pi}}\right)+R^{-}\left(\frac{m_i}{2}+\sqrt{\frac{m_i}{2\pi}}\right)\right)$$

$$=(R^{+}+R^{-})\frac{m}{2}+\frac{R^{+}-R^{-}}{\sqrt{2\pi}}\left(\sum_{i\in C}\sqrt{m_i}-\sum_{i\in N\setminus C}\sqrt{m_i}\right). \tag{22}$$

Thus the best decision in the committee is to accept the proposal if (20) holds, that is, after substituting and simplifying, if

$$\sum_{i\in C}\sqrt{m_i}-\sum_{i\in N\setminus C}\sqrt{m_i}>\frac{m(\Delta^{-}-\Delta^{+})}{\Delta^{+}+\Delta^{-}}\sqrt{\frac{\pi}{2}}.$$

This inequality holds if and only if

$$\sum_{i\in C}\sqrt{m_i}>\frac{1}{2}\sum_{i\in N}\sqrt{m_i}+\frac{m}{2}\frac{\Delta^{-}-\Delta^{+}}{\Delta^{+}+\Delta^{-}}\sqrt{\frac{\pi}{2}} \tag{23}$$

If $\Delta^{-}\geq\Delta^{+}$ this condition defines a weighted majority rule with weights $w_i=\sqrt{m_i}$, and quota that depends on the population figures and the ratio between the marginal utility of winning a negative vote w.r.t. to that of winning a positive one (i.e. $\frac{\Delta^{-}}{\Delta^{+}}$)

$$Q(\tfrac{\Delta^{-}}{\Delta^{+}})=\frac{1}{2}\sum_{i\in N}\sqrt{m_i}+\frac{1}{2}\frac{\frac{\Delta^{-}}{\Delta^{+}}-1}{\frac{\Delta^{-}}{\Delta^{+}}+1}m\sqrt{\frac{\pi}{2}}, \tag{24}$$

or relative quota

$$q(\tfrac{\Delta^{-}}{\Delta^{+}})=\frac{1}{2}+\frac{1}{2}\frac{\frac{\Delta^{-}}{\Delta^{+}}-1}{\frac{\Delta^{-}}{\Delta^{+}}+1}\frac{m\sqrt{\frac{\pi}{2}}}{\sum_{i\in N}\sqrt{m_i}}. \tag{25}$$

Thus, when the importance is the same, i.e. $\Delta^{+}=\Delta^{-}$, then the relative quota is $\frac{1}{2}$, and increases when $\frac{\Delta^{-}}{\Delta^{+}}$ increases. We have then the following result:

Proposition 6 *Assuming that all the represented groups are large enough, if the marginal utility of winning a negative vote is not smaller than the marginal utility of winning a positive vote (i.e. $\Delta^{-}\geq\Delta^{+}$), the weighted majority rule $\mathcal{W}_N=\mathcal{W}_N^{(w,q)}$ in the committee for weights $w_i=\sqrt{m_i}$ and relative quota q given by (25) implements a priori the utilitarian principle with great approximation. In particular, when the marginal utility of winning a negative vote is the same than that of winning a positive vote, i.e. $\Delta^{-}=\Delta^{+}$, the optimal quota is one half.*

Observe that in the case $\Delta^{-}=\Delta^{+}$, the recommendation coincides with the well-known 'second square root rule' recommendation, which is usually derived from other that utilitarian considerations (see Moriss (1987, 2002), and Felsenthal and Machover (1999)).

Now consider the case in which $\Delta^- \leq \Delta^+$, so that (23) may define an *improper* voting rule. In this case the best that can be done seems to be to low the quota Q as much as possible in such a way that $\mathcal{W}_N^{(w,Q)}$, for weights $w_i = \sqrt{m_i}$, is a proper rule. Namely, take

$$\bar{Q} := Min\ \{Q : \sum_{i \in C} w_i > Q \Rightarrow \sum_{j \in N \setminus C} w_j \leq Q\}. \tag{26}$$

The following proposition shows how $W^{(w,\bar{Q})}$ is *almost* the utilitarian optimum.

Proposition 7 *If a voting rule $\mathcal{W}_N$ implements the utilitarian optimum according to the approximation based on (21) and (22), then for all $C \in \mathcal{W}_N$, $\sum_{i \in C} w_i \geq \bar{Q}$.*

Proof. In the proof we use the notation $w(C) := \sum_{i \in C} w_i$. First note that, as $\Delta^- \leq \Delta^+$, and $\frac{w(N)}{2}$ is among the Q's that satisfy (26), we have $Q(\frac{\Delta^-}{\Delta^+}) \leq \bar{Q} \leq \frac{w(N)}{2}$, where $Q(\frac{\Delta^-}{\Delta^+})$ is given by (24). Then obviously

$$w(C) < \bar{Q} \Rightarrow w(N \setminus C) > \bar{Q} \geq Q(\tfrac{\Delta^-}{\Delta^+}).$$

Assume that for some $C \in \mathcal{W}_N$ we have $w(C) < \bar{Q}$. We can assume C to be minimal winning. Let

$$\mathcal{W}'_N := (\mathcal{W}_N \setminus \{C\}) \cup \mathcal{W}^{N \setminus C}.$$

That is, $\mathcal{W}'_N$ is the rule that results from $\mathcal{W}_N$ by eliminating C from the set of winning configurations, and adding $N \setminus C$ and all those containing it. As C is minimal, $N \setminus C$ intersects all $T \in \mathcal{W}_N \setminus \{C\}$, and $\mathcal{W}'_N$ is a proper rule. Let us see that $\mathcal{W}'_N$ is better than $\mathcal{W}_N$ from the utilitarian point of view. In order to compare the aggregated expected utility of a decision made by either rule, note that the decision differs only for the configuration C and for those T containing $N \setminus C$. For all the latter, as $w(T) \geq w(N \setminus C) > \bar{Q} \geq Q(\frac{\Delta^-}{\Delta^+})$, the decision by $\mathcal{W}'_N$ (acceptance) is utilitarian-better than by $\mathcal{W}_N$ (rejection). The reverse only occurs for the configuration C. It is then enough to show that what is lost by rejecting for configuration C, is outweighed by what is gained by accepting for the equally probable configuration $N \setminus C$. Again using (21) and (22), we have

$$E_p[\sum_{k \in M} u_k \mid C \ \&\ Acc] - E_p[\sum_{i \in M} u_i \mid C \ \&\ Rej]$$

$$= (\Delta^+ + \Delta^-)\frac{m}{2} + \frac{\Delta^+ - \Delta^-}{\sqrt{2\pi}}(w(C) - w(N \setminus C)) < (\Delta^+ + \Delta^-)\frac{m}{2}$$

while

$$E_p[\sum_{k \in M} u_k \mid N \setminus C \ \&\ Acc] - E_p[\sum_{i \in M} u_i \mid N \setminus C \ \&\ Rej]$$

$$= (\Delta^+ + \Delta^-)\frac{m}{2} + \frac{\Delta^+ - \Delta^-}{\sqrt{2\pi}}\,(w(N \setminus C) - w(C)) > (\Delta^+ + \Delta^-)\frac{m}{2}$$

Therefore, $\mathcal{W}_N$ does not implement the utilitarian optimum according to the approximation based on (21) and (22). ■

Therefore an utilitarian-optimal (according approximations (21) and (22)) voting rule should contain the winning configurations in $W^{(w,\bar{Q})}$ *plus some* configurations whose weight equals the quota $\bar{Q}$ if such thing is possible. Then we have the following

Corollary 8 *Assuming that all the groups represented are large enough, if the marginal utility of winning a negative vote is smaller than the marginal utility of winning a positive vote (i.e. $\Delta^- < \Delta^+$), the weighted majority rule $\mathcal{W}_N = \mathcal{W}_N^{(w,\bar{Q})}$ in the committee for weights $w_i = \sqrt{m_i}$ and quota $\bar{Q}$ given by (26) implements the a priori utilitarian principle with great approximation.*

8 Concluding remarks

Weighted majority rules are often used in real-world 'committees' of different type. We have examined the rationale of their choice in committees of representatives based on an *a priori* model. In this type of committees, where voters represent groups of different sizes, the idea of egalitarianism may suggest to assign different numbers of votes for different voters. This intuition proves not to be true if the sizes of the represented groups are large enough. Against the traditional voting power claim based on a notion of voting power (in our view lacking solid foundations), our analysis in terms of utilities, in spite of using the same underlying assumption concerning *a priori* voters' preferences, leads to the conclusion that the choice of voting rule hardly matters. It is utilitarianism that motivates the choice of different weights and quota depending of the groups' sizes. If all groups are large, utilitarianism recommends weights proportional to the square root of the size of the represented group, while the quota depends on the ratio between the marginal utility of winning a negative vote w.r.t. the marginal utility winning a positive vote. When this ratio is one, i.e. these marginal utilities are equal, the relative quota recommended is $\frac{1}{2}$ (this is the well-known 'second square root rule'). As this ratio increases the recommended quota increases and is calculated in terms of this ratio, while for a ratio below $\frac{1}{2}$ keeping the quota at $\frac{1}{2}$ 'almost' implements the utilitarian optimum.

This setting allows for a clear comparison with some well-known results in the voting power literature. One the one hand, this setting gives some foundation to the requirement of egalizing Banzhaf indices, and to the "square root rule". On the other hand, these results also show distorsion caused by taking 'voting power' instead of utility as the substantive issue when equality is not achieved. When works in power indices are devoted to finding the rule that would equalise individuals power indices, we show that the differences between individuals are negligible to be cared of.

These results provide additional (utilitarian) support to the 'second square root rule' recommendation and extends it for the case in which marginal utilities of winning a positive and a negative vote differ. It is worth remarking the fact that these conclusions only depend on the marginal utilities of winning a positive vote and that of winning a negative vote, or more strictly speaking, a single parameter: their ratio.

References

[1] Banzhaf, J. F., 1965. Weighted voting doesn't work: A mathematical analysis. *Rutgers Law Review* **19**, 317-343.

[2] Barberà, S., and M. Jackson, 2006. On the Weights of Nations: Assigning Voting Power to Heterogeneous Voters. *Journal of Political Economy* **114(2)**, 317-339.

[3] Beisbart, C., and L. Bovens, 2007. Welfarist evaluations of decision rules for boards of representatives. *Social Choice and Welfare* **29**, 581-608.

[4] Coleman, J. S., 1971. Control of Collectivities and the Power of a Collectivity to Act. In *Social Choice*, edited by B. Lieberman, Gordon and Breach, London.

[5] Felsenthal, D. S., and M. Machover, 1998. *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*. Edward Elgar Publishers, London.

[6] Felsenthal, D. S. and M. Machover, 1999, Minimizing the Mean Majority Deficit: the Second Square-Root Rule, *Mathematical Social Science* **37**, 25-37.

[7] Fleurbaey, M., 2007. Weighted Majority and Democratic Theory, mimeo.

[8] Laruelle, A., and F. Valenciano, 2008. *Voting and Collective Decision-Making: Bargaining and Power.* To be published in Cambridge Univ. Press.

[9] Leech, D., 2002, Designing the Voting System for the EU Council of Ministers, *Public Choice* **113**, 437-464.

[10] Morriss, P., 1987. *Power - A philosophical Analysis*, Manchester University Press, Manchester and New York.

[11] Morriss, P., 2002. *Power - A philosophical Analysis,* 2nd edition, Manchester University Press, Manchester and New York.

[12] Slomczynski, S. and K. Zyczkowski, 2006. Penrose voting system and optimal quota, *Acta Physica Polonica* **B37**, 3133-3143.

[13] von Neumann, J. and O. Morgenstern, 1944, 1947, 1953. *Theory of Games and Economic Behavior.* Princeton: Princeton University Press.