Softest Player is the Most Popular in the Coalition Formation Game

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June 2008

Introduction

The effect of binding agreement reached among a subgroup of participants of negotiation prior to bargaining has been expected to be both positive and negative. Harsanyi's joint bargaining paradox (Harsanyi (1972), Chae and Heidhue (2004), and Vidal-Puga (2005)), received some attention recently which dictates that forming a coalition would worsen the position of the members based on the prediction of the Nash bargaining solution, on the one hand. On the other hand, in some realistic occasions, coalition formation can be observed and the insight given by the coalition formation theory tells that such coalition is formed because it must be beneficial to all of the members.

We try to investigate this problem combining sequential bargaining game to split a dollar a la Binmore and Rubinstein with the coalition formation game utilized by Bloch (1996) and Ray and Vohra (1999) for the class of bargaining problems to split money with CRRA utilities (this name is used for the sake of convenience mostly as what utility function represents in this model is essentially that of time preference). The game is in two stages, and in the first stage, players form coalitions with coalitional agreement, and in the second baragaining stage, players play ordinary seqential bargaining game but their incentives are governed by the agreement they committed to earlier. The solution we adopt here is the limit outcomes of the stationary subgame perfect equilibria when the interval between offers are made vanishes. As is well known, the result often depends upon the protocol of the game in each stage (cf. Baron and Ferejohn (1989), Montero (1999)) as well as the coverage of the coalitional agreement. Especially, we consider the case of predetermined proposer rule which is a kind of deterministic representation of random proposer rule, in the sense that over time the average probability yields equal probability for each player. We compare this result with the one under the fixed order rule (where the player who rejects the standing offer becomes the next proposer). Even though we do not consider the option to let a single representative to bargain on behalf of the members, still we get a possibility of the joint bargaining paradox under the latter rule.

More problematic in the analysis is to have a satisfactory assumption on the type of coalitional contracts. Coalition formation literatures often assume that the choice of actions is determined by noncooperative equilibria under a coalition structure as in Ray and Vohra (1997) and so there is essentially choice of strategies is not much of an issue. Focus is mainly put on the redistribution of payoffs among coalition members. With the assumption of utilities linear in money, the analysis goes well with the restriction of the contracts to those specifying constant share to each member, and in fact Imai and Watanabe (2005) found that players are neutral in forming a coalition. However, with nonlinear utilities, contracts specifying fixed share becomes too restrictive because depending upon the size of the pie to be divided, the share corresponding to the bargaining outcome may vary. However, allowing any sharing contract as an eligible contrat implies that players are bargaining over the shares for off-path events under complete information. This makes the problem intothe one of artificially creating "virtual" utility of a member to the interest of the coalition members, and so the issue resembles that of delegated bargaining with a contract choice as analyzed by Burtraw (1992,1993). To avoid this route, here we assume that signing contract simply implies that they are going to divide the total earning later, and division of the earning is determined by ex post bargaining. (Although ex post bargaining often creates inefficiency as is known in the incomplete contract leterature, here efficiency loss is not of a problem.) As a result, we have a point estimate of the resulting payoffs once a coalition structure is given. The resulting reduced form is known as a hednic game (especially after Bogolomonaia and Jackson (2001), and so the probelm boiled down to find equilibrium coalition structure in a hednic partition game.

One concomitant issue is the fact that the agreement to form a coalition provides an incentive to maximize coalitional gains, and this gives a perfect correlation of interests among members. As is discudded in Imai and Salonen (2000), this causes a de fact delegation to the "toughest" player in the coalition in the bargaining process, where the toughness is represented by the RRA. This provides a room for the positive effect of coalition formation in a pure baragaining model. However, under the fixed order rule, this entails in the Harsanyi's paradox so that no caolition forms. By contrast, under the prefixed proposer rule, a nontrivial coalition formation problem arises. As a modification of the core or its farsighted extension in the sequential coalition formation game, we investigate the order independent equilibrium (first introduced by Mordovanu and Winter (1995) and received more attention these days, cf. Koscy (2006) for example) coalition structure, which exhibits the properties that for generic situations, the softest player joins the coalition which obtains the highest share (per person) and this coalition consists of all players softer than a certain player plus possibly the toughest player. Further the above mentioned structure is nested, which is a remniscate of the top coalition property (Banerjii, Konishi, and Sonmez (2001)) for the non-empty core.

Model

Here, we give definition of the game and the solution. The set of players is $N = \{1, ..., n\}$. Each player's utility function on money is given by u_i which is a continuous, differentiable, concave, and increasing function on R_+ with $u_i(0) = 0$. In particular, we shall make use of CRRA assumption by letting each player's utility function be given by $u_i(x_i) = x_i^{\alpha_i}$ with $\alpha_i \in (0, 1]$.

Fix an ordering o on N, say $1 > 2 > ... > n$. In the first stage, at the outset, player 1 makes a proposal of a formation of the coalition $S_1 \ni 1$, and members of S_1 other than 1 replies by "Yes" or "No". If everybody says "Yes", then S_1 forms. (In particular, if $S_1 = \{1\}$, then S_1 forms automatically.) If somebody rejects the proposal by saying "No", then player 2 makes the next proposal in the next period and the same procedure follows as in the first period. In case S_1 forms, then the first player with respect to o in $N \setminus S_1$ makes the next proposal to members within $N \setminus S_1$ in the same period. The game ends if no player remains to make the next proposal and a coalition structure $\Pi = \left(S^i\right)_{i=1}^k$ results. If the game continues without an end at this stage, then impasse outcome (D) realizes and each player recieves 0 utility.

In the second stage, players engage in a sequential bargaining game with a coalition structure Π. Given a fixed order on N, o, player 1 makes an offer $x = (x_i)_{i=1}^n$ with $x_i \geqq 0$ and $\sum_{i=1}^n x_i = 1$. Player 2,3,$\cdots$,n replies by "Yes" or "No" in this order and if everybody says "Yes", then this stage ends with x in period 0. If some player rejects, then in period 1, player 2 makes an offer, to which players $3, \cdots, n, 1$ respond by "Yes" or "No". The stage ends if everybody says "Yes", whereas if somebody rejects 2's offer, then in period 2, the similar process continues with player 3 making the offer, and so on. This stage ends either with everybody agrees player i's offer x in period $t\left((x,t)\right)$ or perpetual disagreement (D).

Provided the stage 2 ends with agreement (x, t), the game moves into the third stage where within each $S \in \Pi$, essentially the same game as in the second stage is played to decide a division of the coalitional surplus $x(S) = \sum_{i \in S} x_i$ among members, given a fixed order on members of S. We assume that agreement within a coalition and consumption are made independent of whether agreement is reached in other coalitions.

We consider a stationary subgame perfect equilibrium of this game. That is, in an equilibrium strategy, the action choice in stage 3 is independent of history except for the coalition each player belongs to coalitional surplus, and the offer made in that period (if the action is the choice of "Yes" or "No"); and the action choice of stage 2 is independent of history except that the coalition the player belongs to and the offer made in that period (if the player is choosing between "Yes" and "No").

As mentioned above, we shall investigate the solution when δ tends toward 1. It is known that for the third stage, for each $S\,(\neq \phi) \subset N$ and coalitional surplus $x(S)\,(\in [0, 1])$, there is a unique stationary subgame perfect equilibrium, and its limit outcome is given by $\arg\max \left\{ \prod_{i \in S} u_i(x_i) : \Sigma x_i = x(S), \quad x_i \geqq 0. \right\}$.

To begin with, we show that a virtual representation occurs in the second stage. The exact treatment of this situation is given in Imai and Salonen (2000) under a different rule, and we shall describe the result only in the limit when δ tends toward 1 (for detail, see Imai and Yonezaki (2006) which deals with concave utility functions, taht is the case under the CRRA assumption here).

First of all, note that each player is interested in an increase in $x(S)$ with $i \in S \in \Pi$. Therefore for each $i \in S\,(\in \Pi)$, decision over whether to accept an offer x or not depends solely on $\sum_{i \in S} x_i = x(S)$. This implies that an offer x is accepted by members of S only if the player in S whose minimum acceptance level is the greatest, i.e. the toughest player. (Of course the identity of the toughest player may vary as $x(S)$ changes, in general.) Thus, essentially all players in S act in accordance with the toughest player's preference in S, on the one hand. On the other hand, our rule indicates that each player in S has an equal power (up to discounting) to become a proposer. Consequently, the second stage becomes equivalent to a bargaining game with $s\,(=|S|)$ identical players in each coalition such that each player's preference is replaced by the "toughest" player's preference in that coalition. To derive "toughest" player's preference, let $y_j^{\delta}(X)$ stand for j's share in money determined in the third stage given δ, $S \ni j$, and $x(S) = X$. Then let $z_j^{\delta}(X)$ satisfy $u_j\left(z_j^{\delta}(X)\right) = \delta u_j\left(y_j^{\delta}(X)\right)$. Given δ at X, $\arg\max z_j^{\delta}(X)$ yields who is the toughest. As is routine in this literature, in the limit as δ tends toward 1, this condition becomes $\arg\max \frac{u_j(y_j(X))}{u_j'}$, where $y_j(X)$ is the corresponding limit share of j. The quantity u_j/u_j' is called the boldness measure (cf. Aumann and Kurz (1977), Roth (1989), Chae (1993), Burgos, Grant and Kajii (2002)). Thus one may represent the "toughest" player's preference of S by $\left(\arg\max u_j/u_j'\right)' s$ preference, and if there are switches in the identity of the "toughest" player, then one can adjust player's utility level at each switching level to obtain a "surrogate" utility function U_S for each $S \in \Pi$. Then the limit outcome in the second stage can be given as $\arg\max \prod_{S \in \Pi} u_S^s(x)$.

Since we assumed that utilities are of CRRA type, this implies that the magnitude of RRA completely measures the toughness of each player. I.e. since $\frac{u_j'}{u_i} = \alpha_i$, the larger α_i is, the tougher a player is.

Note that the limit outcome in the third stage for $S \in \Pi$ and $x(S)$ is $\left(\frac{\alpha_i}{A_S}\right)_{i \in S}$ where $A_S = \prod_{i \in S} \alpha_i$. In the second stage, the limit outcome is given by

$$\left(\frac{s_i \alpha_{S_i} \cdot \alpha_i}{A_{S_i} \cdot \left(\sum_{S \in \Pi} s \alpha_S\right)}\right)_{i=1}^{n}, \text{ where } i \in S_i \in \Pi,\ \alpha_S = \max_{j \in S} \alpha_j, \text{ and } s_i = |S_i|.$$

This determines the payoffs for the first stage (reduced) game. What makes analysis easier is the fact that the payoffs are proportional to the weights α_j's, and hence players' incentives to form a coalition is aligned at least in some respect. In other words, if we define the coalitional coefficients by $c(S,\Pi) = \dfrac{s\alpha_S}{A_S \cdot \left(\sum\limits_{S'\in\Pi} s'\alpha_{S'}\right)}$, then the preference order of player i over coalition together with a coalition structure is completely reflected in $C(S,\Pi)$(with $i \in S$).

For the analysis of the first stage outcome, we assume the generic α_i's, i.e. all α_i's are distinct. In the first stage, in general the limit outcome depends on the ordering and so unlike the second and the third stage the outcome, in particular the equilibrium coalition structure may not be unique in this sense. By the backward induction procedure over all possible set of remaining players (which is finite) one can assure the existence of the stationary subgame perfect equilibrium. Furthermore, for $3 \leq n \leq 4$, one can confirm that the equilibrium coalition structure is generically unique. However, in general, there could be a dependence of the outcome on the ordering. Below, we shall concentrate on this reduced hedonic game.

Hedonic game

Since the reduced game for the first stage is a hedonic partition function game, one may assiciate the equilibrium coalition structure with the stable coalition structure discussed in the literature, mainly for the hedonic characteristic finction game (cf. Banerjii, Konishi, and Sonmez (2001), Bogomolnaia and Jackson (2001)). Several concepts have been proposed by far and one of the major concerns is the non-emptiness of the solution.

Among several criteria proposed, the core is the most basic concept, in which the absense of a coalition (currently not in the structure) which foresees an immediate gain by deviation to form. Nash stability and seberal derived concepts restrict the deviation to a single player. Farsighted stability requires a lack of deviation by a coalition when taking into account of a chain of reactions by other coalitions.

Compared to these stability concepts, equilibrium coalition structures we consider lies somewhere between the core and the farsighted stable set, in the sense that players foresee the consequence of their deviation but along with a certain ordering and they can assume that the formed coalition as a guaranteed thing. Especially order dependence is one property which helps existence while potentially producing too many solutions. Backed by this motivation, here we shall focus upon the equilibrium coalition structures which is independent of orderings. We call this property, order independence (OI).

First we make a preliminary observations:

Lemma :(a) $\Pi = \{N\}$ is not an equilibrium coalition structure except for the case where $\alpha_i = \alpha$ for all i(which we preclude here).

(b) Given an equilibrium coalition structure Π, for $i, j \in N, \{i\}, \{j\} \in \Pi$ only if $\alpha_i = \alpha_j$ (which we preclude here)

(c) Over coalitions including two players, those two players' preferences coincide.

Proof: We provide proof for (b). In Π, for generic $(\alpha_i)_N$ at most one $S \in \Pi$ with $s = 1$.

(If there are $\{i\}$ and $\{j\}$ with $\{i\}, \{j\} \in \Pi$, then defining $\Pi' = (\Pi \setminus \{i\}, \{j\} \cup \{i,j\})$, $c(\{i\}, \Pi) = c(\{j\}, \Pi) = \frac{1}{(\alpha_i+\alpha_j+A^C)} < \frac{2\alpha\{i,j\}}{\left(2\alpha_{\{i,j\}}+A^C\right)(\alpha_i+\alpha_j)} = c(\{i,j\}, \Pi')$.)

These properties show that there is always a benefit from forming a coalition from a single player, but to include all is not beneficial. This can be seen from the formula for C saying that the gain from coalition stems from the size s and the toughness of the toughest players in the coalition, and the property of the game where players are dividing fixed amount of money. (The effect of the size consisting of softer players is observed by Harrington (1986) for the majoritarian bargaining.) This suggests that the softest player would be popular because it contribute to the size of coalition while demanding modestly inside the coalition. Then the toughest player is also polular? Not always, as the next example shows.

Example: Consider 3 person case, with $\alpha_1 > \alpha_2 > \alpha_3$.

Then the equilibrium coalition structure is either $\Pi = \{\{1,3\},\{2\}\}$ or $\Pi' = \{\{2,3\},\{1\}\}$ depending upon the sign of $[C(\{1,3\},\Pi) - C(\{2,3\},\Pi')]$ or that of $[\alpha_2^2(\alpha_1 - \alpha_3) \geq \alpha_1^2(\alpha_2 - \alpha_3)]$. Thus roughly speaking, if α_2 is close to α_3, then $\{1,3\}$ forms, while if α_2 is close to α_1, then $\{2,3\}$ forms. In the former case, player 1's toughness is unsurmountable, so to have 1 as the toughest player to obtain higher share for the coalition while having 3 to inflate the size of coalition is the optimal for the participants. In the latter case, 1 and 2 are close substitutes, and so 2 may be chosen as the representative by 3 to protect its internal share. In either case, the softest player 3 belongs to the formed coalition. Further, the equilibrium coalition structure does not depend on the ordering.

In the case of 4 person problem, by Lemma 1, either one 3 person coalition, or two 2 person coalitions form. Let S be the coalition with the highest C under this restriction. Then all the member may wait to form S in any ordering. Thus order independent equilibrium coalition structure exists.

For 5 players game, situation becomes a bit more complicated, as even if 2 players coalition is the best to form with the expectation of remaining 3 players partitioned into 2 players and one player, under the ordering with one of those three players as the initial proposer, they may choose different two players coalition to possibly induce different best response. Although we do not have exact counterexample by far, we shall concentrate on the equilibrium coalition structures satisfying (OI), which allow us to avoid these issues.

We list further properties one derive from the formula for C. Thoroughout, we shall assume that $\alpha_1 > \alpha_2 ... > \alpha_n$.

1. Representation Effect : Suppose $S\,(\neq \phi, N)$ forms. Then for any $\Pi \ni S$, $c(S,\Pi) \geqq \frac{s\alpha_S}{\left(s\alpha_S+(n-s)\alpha_{N\setminus S}\right)A_S}$.

(For, $(n-s)\,\alpha_{N\diagdown S}$ is the upperbound for possible representative coefficients for any subpartition of $N\diagdown S$.)

2. Replacement : If $c\left((s\diagdown\{i\})\cup\{j\},\Pi'\right)>c\left(S,\Pi\right)$ where $\Pi'=(\Pi\diagdown\{S,S'\})\cup\{S\diagdown\{i\}\cup\{j\},(S'\diagdown\{j\})\cup\{i\}\}$.

(For $c\left(S,\Pi\right)=\frac{s\alpha_S}{(s\alpha_S+s'\alpha_{S'}+A^C)A_S}<\frac{s\alpha_S}{(s\alpha_S+s'\alpha_{S'}+A^c)(A_S-\alpha_i+\alpha_j)}=c\left((S\diagdown\{i\})\cup\{j\},\Pi'\right)$.)

3. Redundacy of an above average player : If $i\in S$ and $\alpha_{S\diagdown\{i\}}=\alpha_S$ and $\alpha_i\geqq\frac{A_S}{s}$, then for $\Pi'=(\Pi\diagdown S)\cup\{S\diagdown\{i\},\{i\}\}$, $c\left(S,\Pi\right)<c\left(S\diagdown\{i\},\Pi'\right)$.

(For $c\left(S,\Pi\right)=\frac{s\alpha_S}{(s\alpha_S+A^C)A_S}<\frac{(s-1)\alpha_S}{((s-1)\alpha_S+\alpha_i+A^C)(A_S-\alpha_i)}=c\left(S\diagdown\{i\},\Pi'\right)$.)

4. Addition of the softest player : Let $\alpha_n=\min_{i\in S\cup S'}\alpha_i$ with $n\in S'\neq\{n\}$ and $\alpha_S>a_{S'}$. Then for $\Pi'\left(\Pi\diagdown\{S,S'\}\right)\cup\{S\cup\{n\},S'\{n\}\}$, $c\left(S,\Pi\right)<c\left(S\cup\{n\},\Pi'\right)$:

(For, $c\left(S,\Pi\right)=\frac{s\alpha_S}{(s\alpha_S+s'\alpha_{S'}+A^C)A_S}$ and $c\left(S\cup\{n\},\Pi'\right)=\frac{(s+1)\alpha_S}{((s+1)\alpha_S+(s'-1)\alpha_{S'}+A^C)(A_S\{\alpha_n\})}$, and $c\left(S\cup\{n\},\Pi'\right)=\frac{(s+1)\alpha_S}{((s+1)\alpha_S+(s'-1)\alpha_S)}$, and $c\left(S,\Pi\right)<c\left(S\cup\{n\},\Pi'\right)$ is equivalent to $(A_S-\alpha_n)\left(s\alpha_S+s'\alpha_{S'}+A^C\right)>(\alpha_S-\alpha_{S'})\,A_S$ which is true because $A_S\geqq\alpha_S$, $\alpha_{S'}\geqq\alpha_n$, $A^C\geqq 0$ and $s\alpha_S\geqq A_S$.)

Next, we define the properties which hold for order independent coalition structures.

(PS) Popularity of the softest player : If n is the only softest player, then in equilibrium, $n\in S^*$ such that $c\left(S^*,\Pi\right)=\max_{S\in\Pi}c\left(S,\Pi\right)$.

(HB) Heavy bottom with or without the toughest player : S^* given as above consists of either, all players i with $i\geqq k$ for some $k\geqq 2$, or all players i with $i\geqq k$ for $k\geqq 3$ and player 1.

Theorem : If sspe coalition structures satisfy (OI), then (PS) and (HB) hold.

Proof : (PS) : Suppose this is not the case and so in Π there are S,S' with $n\in S'$, $c\left(S',\Pi\right)<c\left(S,\Pi\right)$.

Consider the ordering under which all $S''\in\Pi\diagdown\{S,S'\}$ form (such ordering and spe exist vecause of the order-independence), and so there is (on-the^path) subgame consisting of only players in $S\cup S'$. In particular an $i\in S$ could be the initial proposer. Since a singleton coalition is always the worst in terms of the weight, $|S|\,1$. Suppose that there is $i\in S$ with $\alpha_i\leqq\alpha_{S'}$. Then by property 3, we have $c\left(S\cup\{n\}\diagdown\{i\},\Pi'\right)>c\left(S,\Pi\right)$ where Π' is as in property 3. For any subgame equilibrium Π'' (which is a refinement of Π' on the part of S'), $c\left(S\cup\{n\}\diagdown\{i\},\Pi''\right)\geqq c\left(\cup\{n\}\diagdown\{i\},\Pi'\right)$, and n also wishes to join S. Therefore this cannot be an equilibrium.

Next, suppose for all $i \in S$, $\alpha_i > \alpha_{S'}$. In particular $\alpha_S > \alpha_{S'}$ and so by property 5, both S and n have an incetive to werge, and so again this cannot be an equilibrium. This establishes (PS).

As for (HB), note that if a sspe satisfies (OE), then each player proposes the coalition to which that player belongs, which is accepted by the members along the equilibrium path. Thus there would be no delay when $\delta < 1$. So, for any $S_1, S_2 \in \Pi$, there is a subgame reacheable in a SSPE such that $S_1 \cup S_2$ are the remaining players. We refer to such a subgame as a subgame with $S_1$and S_2.

(HB): Suppose there is $j \notin S^*$ but $x(S^*, \Pi^*) < x(S^* \cup \{j\}, \Pi^{*\prime})$ where $\Pi^{*\prime} = (\Pi^* \setminus \{S^*, S^j\}) \cup \{S^* \cup \{j\}, S^j \setminus \{j\}\}$ (where if $S^j \setminus \{j\} = \phi$, cross this term out). We may choose j so that a_j is the smaalest among thse j's. Then in the subgame with S^* and S^j, players in S^* can offer $S^* \cup \{j\}$ and the acceptance by j does not hurt the members of S^* because whatever a subpartition of $S^j \setminus \{j\}$ may be, the sum of weights is at most $(s^j - a)A_{S^j}$.

By (HB), no player whose coefficient is less than the average of S^* is not left outside of S^*.

Next, suppose there is $j \in S^*$ with j's coefficient is more thanthe average of S^*. Then for the subgame with S^*, a member in $S^* \setminus \{j\}$ can propose $S^* \setminus \{j\}$ which shall be accepted.

Finally, suppose $A_{S^*} < a_1$ and there is $j \notin S^*$ with $a_j < A_{S^*}$. Let $i \in S^*$ be $a_i = A_{S^*}$, and $1 \in S^1 \in \Pi^*$. Consider the subgame with S^* and S^1. A member of $S^* \setminus \{i\}$ can propose $(S^* \setminus \{i\}) \quad \cup \{1\}$, and this shall be accepted because whatever a subpartition of $(S^1 \setminus \{1\}) \quad \cup \{i\}$ may be, coalitional weight for S^1 was $s^1 a_1$ and that for a subpartition of $(S^1 \setminus \{1\}) \quad \cup \{i\}$ is at most $s^1 A_{(S^1 \setminus \{1\}) \quad \cup \{i\}} < s^1 a_1$.

Conclusion

Here, we investigated the coalition formation for pure bargaining game under the prtocol where the next proposer is predetermined by an ordering. In this case, there is a possible gain from formation of a coalition, and in fact a nontrivial coaltion structure arises in some equilibrium. If the protocol is the so-called fixed order rule so that the rejecting player get to make the next counter proposal, then the result differs completely. There the "toughest" player to whom a de facto delegation occurs in a formed coalition finds no merit as the size of the coalition does not affect the equlibrium cutcome in the second stage. Thus no coalition formed which is one form of the Harsanyi's joint bargaining paradox.

When coalition does matter, we found that the softest player enjoys the popularity in forming coalition when the equilibrium coalition structure is order independent. To investigate conditions allowing such eqiulibrium outcomes is the future reaserch aggenda.

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